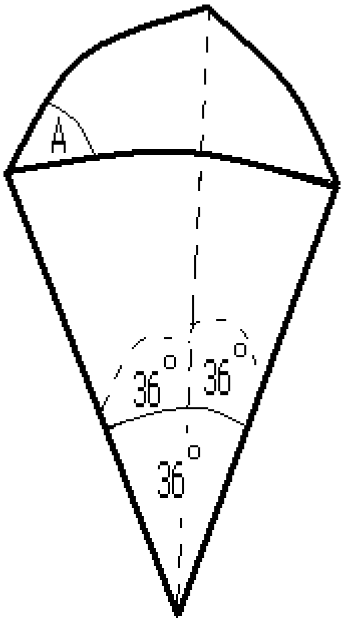
<u>Using spherical trigonometry and the golden ratio to find the angles of the major node connector.</u>



The pentagonal cone has a flat base which contains this star shape where the length of the star arms to the length of one side of the smaller inside pentagon is the golden ratio. However this does not prove that the diagonal to edge ratio of the pentagon *on the surface of a sphere* is equal to the golden ratio as well.

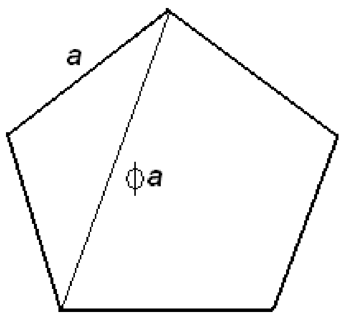
$$\cos(36)=\cos(36)\cos(36)+\sin(36)\sin(36)\cos(A)$$

$$\cos(A)=\frac{\cos(36)-(\cos(36))^2}{(\sin(36))^2}$$

Apply the spherical trigonometry law of cosines to the triangular cone formed by three major disks.
http://en.wikipedia.org/wiki/Spherical_trigonometry

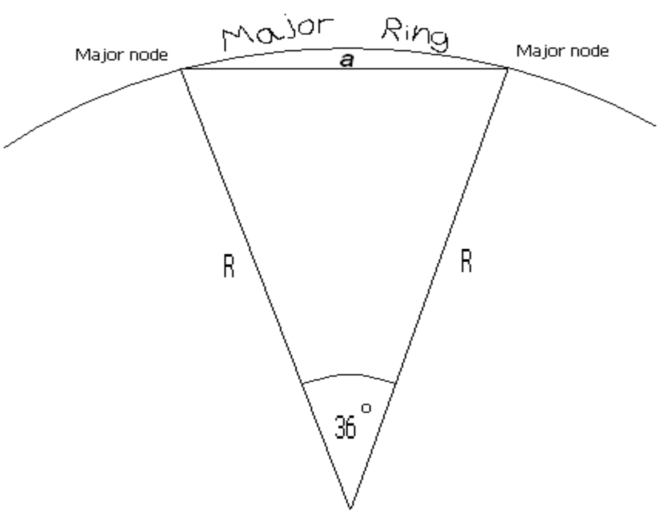
A = 63.43494882 degrees

The definition of the Golden ratio, ϕ (*phi*) is the ratio of the diagonal to side in a regular pentagon *on a flat surface;* and is derived to be Use the sine rule to find the length of the geodesic line connecting two major nodes, this line is also a side of a regular *flat* pentagon, *a.*



$$\phi=\frac{1+\sqrt{5}}{2}$$

Applying this definition find the length of the geodesic connecting two major nodes in a minor ring, this is the diagonal of the flat regular pentagon. Just to check apply the sine rule to this 60 degree segment and see if the Golden ratio is returned.



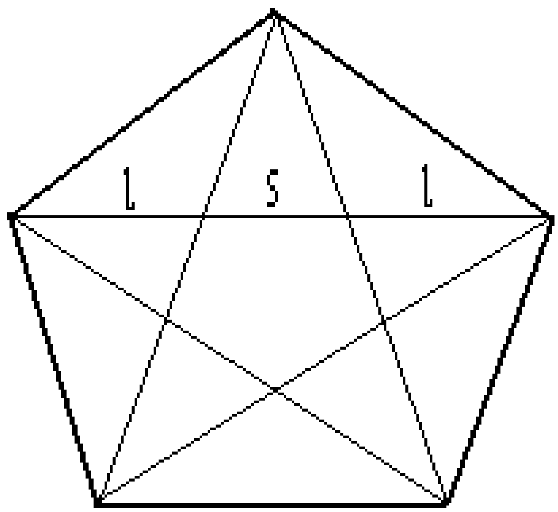
$$\sin(18)=\frac{a/2}{R}$$

$$R\sin(18)=\frac{a}{2}$$

$$a=2\text{R}\sin(18)$$

Find an equation for the length of one of the star arms in terms of the Golden ratio.

Find the length of the edge of the smaller flat pentagon, s. And work through the mathematics backwards to check. diagonal in the regular pentagon = geodesic of minor ring.



$$s = 2\phi R \sin 18° - 2(2\phi R \sin 18° - 2R \sin 18°)$$
$$s = 2\phi R \sin 18° - 4\phi R \sin 18° + 4R \sin 18°$$
$$s = -2\phi R \sin 18° + 4R \sin 18°$$
$$s = 2(-\phi R \sin 18° + 2R \sin 18°)$$
$$s = 2R \sin 18°(-\phi + 2)$$
$$s = 2R \sin 18°(2 - \phi)$$
$$s = 2R(2 - \phi)\sin 18°$$

$$2\phi R \sin 18° = 2l + s$$
$$2\phi R \sin 18° = 4R \sin 18°(\phi - 1) + 2R \sin 18°(2 - \phi)$$
$$2\phi R \sin 18° = 2R \sin 18°\left[2(\phi - 1) + (2 - \phi)\right]$$
$$2\phi R \sin 18° = 2R \sin 18°\left[2\phi - 2 + 2 - \phi\right]$$
$$2\phi R \sin 18° = 2R \sin 18°\phi$$
$$2\phi R \sin 18° = 2\phi R \sin 18°$$
$$correct$$

Q. Find a numerical value for l in terms of R, i.e. l = a value multiplied by R. Then work the algebra backwards to check.

$$l = 2R(\phi - 1)\sin 18°$$
$$l = 2R\left(\frac{1+\sqrt{5}}{2} - 1\right)\sin 18°$$
$$l = \left(\frac{2R + 2\sqrt{5}R}{2} - 2R\right)\sin 18°$$
$$l = \left(R + \sqrt{5}R - 2R\right)\sin 18°$$
$$l = \left(\sqrt{5}R - R\right)\sin 18°$$
$$l = R\left(\sqrt{5} - 1\right)\sin 18°$$

$$R(\sqrt{5} - 1)\sin 18°$$
$$\frac{2}{2}R(\sqrt{5} - 1)\sin 18°$$
$$2R\left(\frac{\sqrt{5} - 1}{2}\right)\sin 18°$$
$$2R\left(\frac{\sqrt{5} - 1 + 1 - 1}{2}\right)\sin 18°$$
$$2R\left(\frac{\sqrt{5} + 1}{2} - \frac{1}{2} - \frac{1}{2}\right)\sin 18°$$
$$2R\left(\frac{1+\sqrt{5}}{2} - 1\right)\sin 18°$$

$$l = 0.3819660113R$$

R will cancel out at the next stage so leave it there,

Q. Find a numerical value for s in terms of R and as before work the algebra backwards to check.

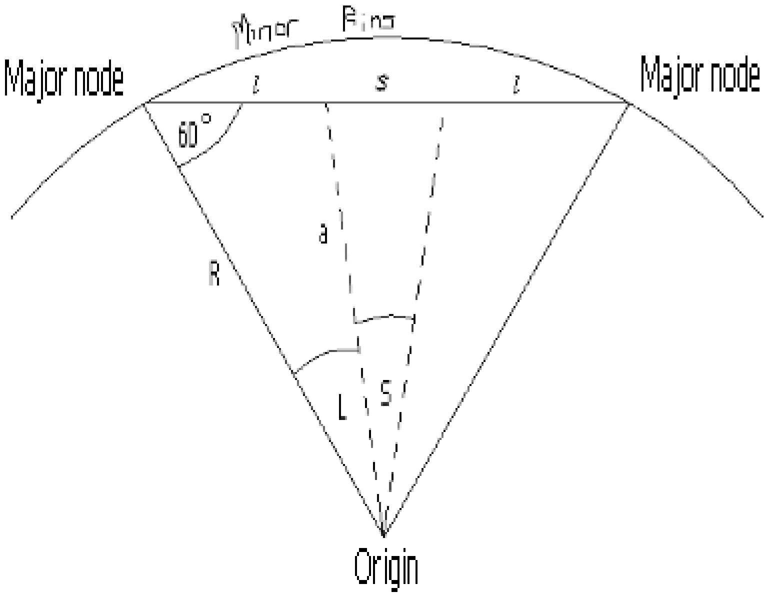
$$s = 2R(2 - \phi)\sin 18°$$
$$s = 2R\left[2 - \left(\frac{1+\sqrt{5}}{2}\right)\right]\sin 18°$$
$$s = \left(4R - \frac{2R - 2\sqrt{5}R}{2}\right)\sin 18°$$
$$s = \left(4R - R - \sqrt{5}R\right)\sin 18°$$
$$s = \left(3R - \sqrt{5}R\right)\sin 18°$$
$$s = R\left(3 - \sqrt{5}\right)\sin 18°$$

$$s = 0.2360679775R$$

$$s = R(3 - \sqrt{5})\sin 18°$$
$$s = 2R\left(\frac{3 - \sqrt{5}}{2}\right)\sin 18°$$
$$s = 2R\left[-\left(\frac{\sqrt{5} - 3}{2}\right)\right]\sin 18°$$
$$s = 2R\left[-\left(\frac{\sqrt{5}}{2} - \frac{3}{2}\right)\right]\sin 18°$$
$$s = 2R\left[2 - \left(\frac{\sqrt{5}}{2} - \frac{3}{2} + \frac{4}{2}\right)\right]\sin 18°$$
$$s = 2R\left[2 - \left(\frac{\sqrt{5}}{2} + \frac{1}{2}\right)\right]\sin 18°$$
$$s = 2R\left[2 - \left(\frac{\sqrt{5} + 1}{2}\right)\right]\sin 18°$$

correct.

Q. Below is a drawing of part of a minor disk, find the internal angles S and L on this disk using the cosine and then the sin rule. The segment formed by the origin, two major nodes including the diagonal of the regular pentagon has angle at origin 60 degrees. It is Symmetric so it is an isosceles triangle The sum of angles in triangle = 180 degrees ∴ it is an equilateral triangle.



$$a^2 = R^2 + l^2 - Rl$$

$$l = 0.3819660113R$$

$$a^2 = R^2 + 0.1458980338R^2 - 0.3819660113R^2$$

$$\frac{a^2}{R^2} = 0.7639320225$$

$$a = 0.8740320489R$$

Now use the sine rule to find L, then calculate S.

$$\frac{a}{\sin 60°} = \frac{l}{\sin L}$$

$$\sin L = \frac{l \sin 60°}{a}$$

$$\sin L = \frac{l\sqrt{3}}{2a}$$

$$\sin L = \frac{0.3819660113R \times \sqrt{3}}{2 \times 0.8740320489R}$$

$$\sin L = 0.3784669791$$

$$L = 22.2387561°$$

$$60° = 2L + S$$

$$60° - 2L = S$$

$$S = 60° - 2 \times 22.2387561°$$

$$S = 15.5224878°$$

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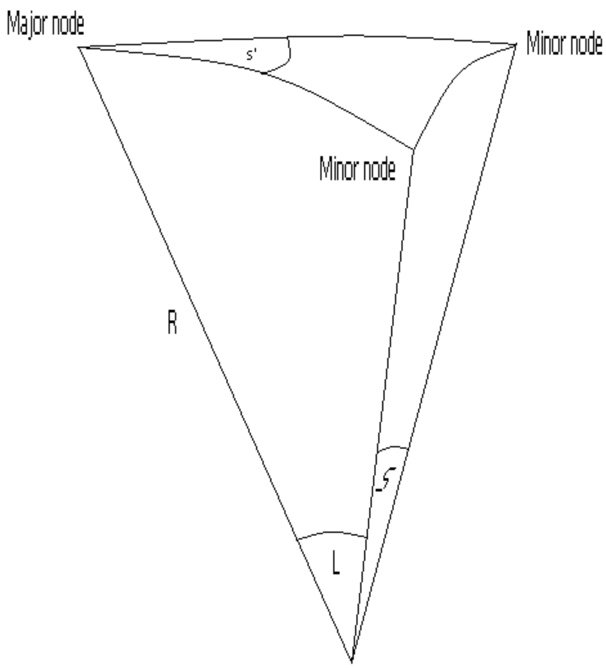
Q The triangular pyramid below is formed of three minor rings forming the 'point of the star'. Use spherical trigonometry law of cosines to find the angle of the 'point of the star' on the surface of the sphere, s'.



$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$b = c = L, a = S, A = s'$$

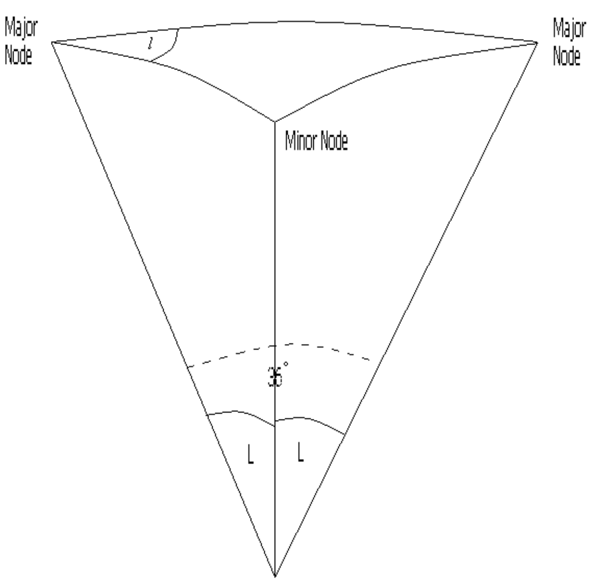
$$\cos S = \cos^2 L = \sin^2 L \cos s'$$

$$\cos s' = \frac{\cos S - \cos^2 L}{\sin^2 L}$$

$$\cos s' = 0.7453559931$$

$s' = 41.81031484°$

Q The triangular pyramid below is formed of two minor rings and one major ring. Use spherical trigonometry law of cosines to find the angle formed by a major and minor ring, l'.



$$\cos a = \cos b \cos c + \sin b \sin c \cos A$$

$$a = L, b = L, c = 36°, A = l'$$

$$\cos L = \cos L \cos 36° + \sin L \sin 36° \cos l'$$

$$\cos L - \cos L \cos 36° = \sin L \sin 36° \cos l'$$

$$\cos L(1 - \cos 36°) = \sin L \sin 36° \cos l'$$

$$\cos l' = \frac{\cos L(1 - \cos 36°)}{\sin L \sin 36°}$$

$$\cos l' = 0.79465...$$

$l' = 37.37736817°$

On the surface of the Conradi sphere there are 3 angles of the major node connector formed between a major ring and a major ring, a major ring and a minor ring, and a minor ring and a minor ring.

List of variables.

R = radius of sphere
l = straight length of an arm of the star in the plane formed by five major nodes. Or one side of the geodesic contained in the sphere.
s = straight length of a side of the small pentagon in this plane.
L = Internal angle opposite the large length, or the arm of the star.
S = Internal angle opposite the small length, or a side of a small pentagon.
s' = Spherical angle of the point of a star, which is what we will compare to the previous calculated value.