The Meaning
of the
Monas Hieroglyphica
with regards to
GEOMETRY

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by
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Where to start in decoding Dee's Monas Hieroglyphica

To the modern reader, the Monas Hieroglyphica just sounds downright weird.

As Gerald Suster writes in "*John Dee, Essential Readings*":

"What is one to make of the Hieroglyphic Monad?
Even Frances Yates confessed that the explanatory text
'leaves the reader thoroughly bewildered.'
Commentators agree that the key is no longer with us,
that key being Dee's oral explanation;
or perhaps we are too far removed from sixteenth-century intellectual sensibilities
to perceive implications deeply significant to intelligent men of that time…
Certainly Dee regarded it as his masterpiece, the summary and crowning synthesis
of all the knowledge and wisdom he had acquired."

(Suster, page 31)

The best way to understand the Monas Hieroglyphica is to read the original text in Latin and contemplate its illustrations. As most people don't read Latin, I've provided an English translation (included in *The Works of John Dee, Modernizations of his Main Mathematical Masterpieces*, by Jim Egan).

But let me emphasize: translations are only approximations. To grasp subtle word and letter clues, you should always return to the primary text (which is also included in the book of *Modernizations*.

THE WORKS OF
JOHN DEE
MODERNIZATIONS OF HIS MAIN
MATHEMATICAL MASTERPIECES
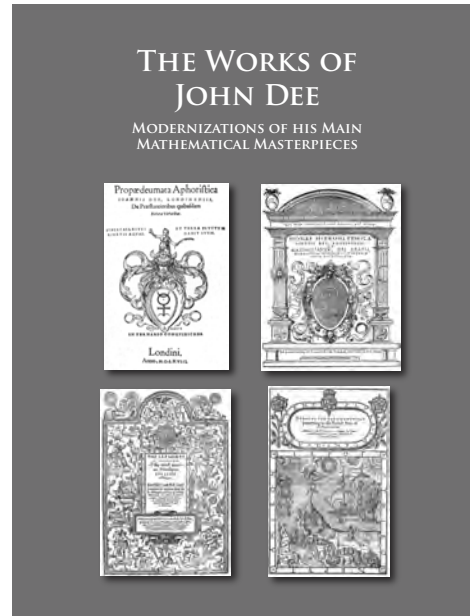
The text seems to be a strange mixture of alchemy, astrology, astronomy, mathematics, Greek philosophy, Hermeticism, Cabala, Christianity, and more. He expresses this in the title of his work, just before Theorem 1.

MONAS HIEROGLYPHICA:
IOANNIS DEE, LONDINENSIS,
Mathematicè, Magicè, Cabalisticè, Anagogiceque,
explicata: Ad
SAPIENTISSIMVM,
ROMANORVM, BOHEMIAE, ET HVNGARIAE,
REGEM,
MAXIMILIANVM.

"Anagogical" is a Greek word (anagogikos) meaning "mystical, spiritual, or allegorical." (OED, p.302)

"Cabbalistically" refers to the ancient Jewish tradition of the mystical interpretation of the Bible, which was a great influence in the Middle Ages.

"Magically," of course, means producing a result using some mysterious unexplained power.

MONAS HIEROGLYPHICA:
JOHN DEE OF LONDON
Mathematically, Magically, Cabalistically, and Anagogically
Explained To
MAXIMILLIAN
Most Wise
KING
of The Romans, Bohemia, and Hungary

"Spiritualism, mysticism and magic" is a mix that might immediately spark skepticism. In modern terms, one might say that Dee is "really out there" and immediately write him off as an irrational, nonsensical nutcase.

However, this depiction does not fit Dee. He was one of the most intelligent, well-read, well-traveled men who lived in Europe during the 1500's. He had the largest library (over 4000 books) in all of England. He tutored the highest courtiers in Elizabeth's court. He wrote about 50 books and treatises on scientific topics like geometry, astronomy, optics, navigation, and mathematics.

Which brings us to the first of the four words he uses in the phrase "Mathematically, Magically, Cabalistically, Anagogically." Mathematics, "to use a modern phrase is an "exact science", a field of science capable of "accurate quantitative expression."

Mathematicè,

In simple terms,
4 x 7 = 28 in Beijing is the same as 4 x 7 = 28 in Peoria.
A tetrahedron to Plato is a tetrahedron today.
Arithmetic and geometry don't change.

Physics, chemistry, and astronomy are also considered "exact sciences," as experiments and measurements are reproducible. Dee's word "Mathematically" is drastically different than "Magically, Cabalistically, and Anagogically," yet it is the very first word he uses in his description.

The Monas Hieroglyphica is not an alchemical text with a few numbers thrown in. It is primarily a book about numbers that are camouflaged in alchemical language.

Dee discovered an interconnection between the two branches of mathematics: arithmetic and geometry. He found that certain natural rhythms found in the realm of number were also found in the realm of shape. He was so astounded by what he found he wanted to share with the world, but he feared the wrong people might use its power, so he disguised his findings in alchemy-speak.

Dee fell gravely ill in 1558 when an epidemic of influenza swept across England. Sensing he might not have long to live, he asked his friend, the Portuguese mathetician Pedro Nuñez to look after his literary affairs should he die. This brush with death seemed to promt Dee into somehow sharing his discoveries with the world, lest they be lost, perhaps forever.

In short, most of the descriptions in the Monas are metaphorical. The "Sun" means much more than "that star 93 million miles away" and the "Moon" mean more than the "sphere dotted with craters that revolves around the Earth." What Dee calls the "Lunar Mercury Planets" and "Solar Mercury Planets" are not really strange categories of astrological objects, they're numbers. Believe it or not, even "point, line, and circle" don't actually mean "point, line, and circle." Dee even informs the reader up front that he is using Gematria, Notaricon and Tzyruph, so we should creatively look for these things in the Monas.

Gematria: letters stand for numbers
Notaricon: letters symbolize concepts or the first letters of a sequence of words form a new word or sentence
Tzyruph: rearranging the letters of a word or phrase to make a new word or phrase, like an anagram of jumbled letters (also called Temurah).

The first step is to read Dee's "preparatory" text for the ideas in the Monas, the *Propadeumata Aphoristica* (meaning "Preparatory Aphorisms"). This text, printed in 1558, consists of 120 Axioms that explain what was known about astronomy and physics at the time. In 1978, Wayne Shumaker and J. T. Heilbron translated and provided an insightful analysis of this work. Their translation features the Latin and English on facing pages. To get a better feel for the book (and because Shumaker and Heilbron's Latin version omits a few important clues) I have reprinted the original Latin version in its original size. With the assistance of Scott Barker, I have made a fresh translation.

original Latin
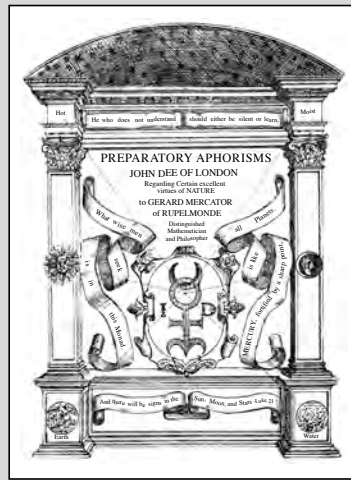
modern English Translation

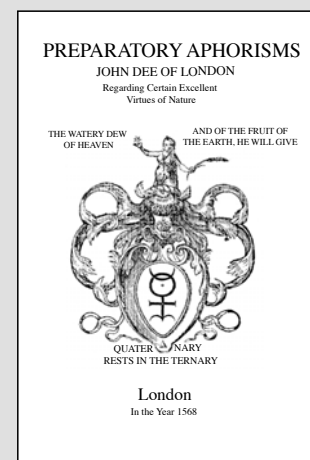Title Page of Dee's
1558 *Propaedeumata Aphoristica*

Even if you find most of the Aphorisms confounding, don't despair. Read the first 20 Aphorisms to get a feel for it and skim through the rest. Particularly note aphorism 9. For a real head-scratcher, read Axiom 18 carefully. Schumacher and Heilbron called it the "most inscrutable of all the Aphorisms." But I will show that it's actually clear statement that helps illuminate the Monas Hieroglyphica.

The 1558 Title Page is quite similar to the Title Page of the *Monas Hieroglyphica*. I consider it a "dry run" for the Monas, and in that sense, it provides a few important clues.

When the *Propaedeumata Aphoristica* was reprinted in 1568, Dee got rid of the "architectural" motif, as it now paled in comparison to the Title Page of the Monas. He replaced it with the design similar to the emblem on the last page of the Monas. The most important feature is that it is emblazoned with that three-word epithet "QUATERNARIUS INTERNARIO CONQUIESCENS." or the "QUATERNARY RESTS IN THE TERNARY," a cryptic, but key theme in the Monas.
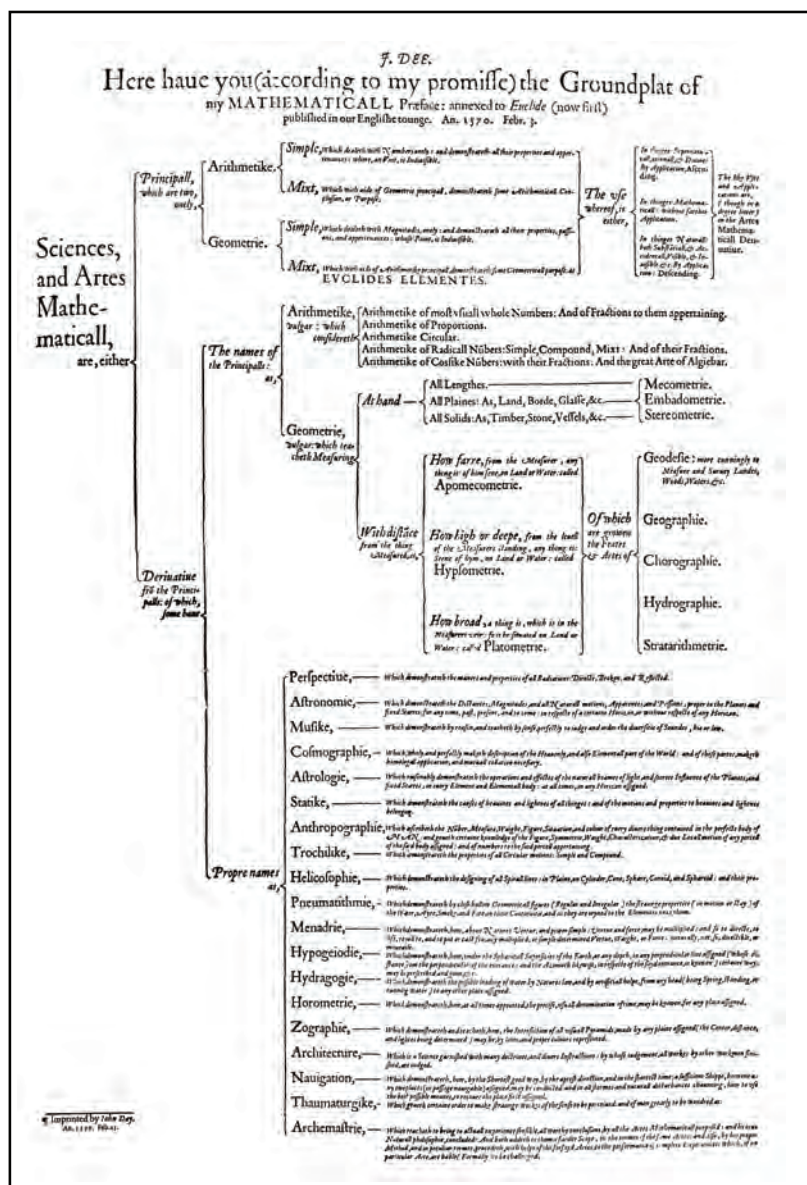

original Latin

modern English Translation

Title Page artwork on Dee's
1568 *Propaedeumata Aphoristica*

Also, study the numbers in the only diagram in the whole book, the one that illustrates the ideas in Aphorisms 116 and 117. It's a chart of various permutations of the conjunctions of the 7 planets. Don't worry if you don't follow the math.

Shumaker and Heilbron noted that Dee "slipped in computing" in that chart. The number 120 in the final column in the chart should actually be 126, not 120. This mistake throws off the total, which should be 25,341 instead of 25,335. (Shumaker p. 91)

These authors are absolutely right. However, Dee didn't "slip in computing," his error was intentional. (Sorry, but my explanations about this chart won't be understandable until after I explain the *Monas*.)

Next, browse through the original printing of Dee's 1570 *Preface to the Euclid* (The *Elements of Euclid* translated by Henry Billingsley) It's in English, but Dee's Elizabethan English is a little hard to trudge through, so I have made a modern transliteration of the work. The first few pages will give you a good feeling for how vitally important mathematics was in Dee's mind.

The "Groundplat" at the end of the Preface is a nice summary of the 19 "Mathematical Arts and Sciences" that Dee describes throughout the text. (Dee was 31 years old when he wrote the *Propadeumata Aphoristica*, 37 when he wrote the *Monas*, and 43 when he wrote the *Preface*.)

Billingsley's translation is quite long, the 15 Books take up over 450 pages. Dee supplemented the text with comments and lemmas in Books 10-15. A sixteenth book was added which was an exposition of two Archimedian solids, (the cuboctahedron and the rhombic dodecahedron) borrowed from a contemporary work by the French mathmetician Francois de Foix, the Count of Candide.
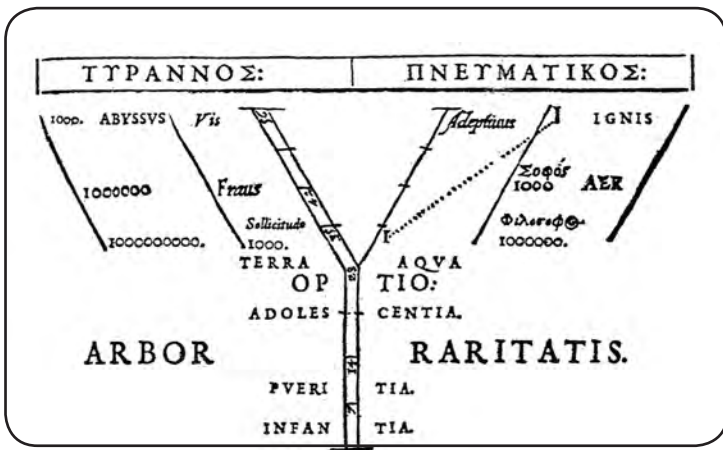
Now for the *Monas Hieroglyphica*. Read or at least browse the introductory "Letter to Maximilian." Again, parts of it might seem unintelligible, but press on. There are a half a dozen important ideas in the "Letter to Maximilian" that you should particularly note.

1 Dee claims to be giving the King a gift that is "extremely rare and of great goodness." (We will discover later what this gift is.)

2 Dee appears to be claiming that he is "1" among 1,000,000,000,000 men of the common sort" who understands not only the "Causes of Celestial powers," but also of "Supercelestial virtues" as well. (A billion and other large multiples of 10 can be found in his "Tree of Rarity" chart.) Pythagoras introduced the metaphorical idea of the letter Y splitting into a "path of vice" and a "path of virtue."


Dee encourages the King to "study it with great attention, as "still greater mysteries will present themselves based on our COSMOPOLITICAL Theories." In Greek, "cosmo" means "world" and "politicos" means "citizen", so a "cosmopolite" is "citizen of the world." While most Elizabethans never left England, Dee was quite worldly. This is one reason he dedicated his work he dedicated his work to the Holy Roman Emperor rather than Queen Elizabeth.

But beyond Dee's awareness if the cultural currents on the continent, Dee thought globally – he coined the phrase "The British Empire." Beyond the globe, Dee thought Celestially– he had studied movements of the Sun, Moon, planets, and stars. But even beyond that, he thought "Supercelestially." I say that because he studied numbers. In the Preface to Euclid, Dee describes numbers as being part "supernatural" and part "natural." (Dee, Preface, p.1 verso)

Dee seems to have stylistically borrowed from a similar chart by French author Geofroy Tory in his 1529 "Champs Fleury," meaning "Field of Flowers," a book about the history of classical letterforms and how to geometrically construct them. But Dee's version is clearly his own construction. (Be sure to study Dee's original Latin version of the "Tree of Rarity" as the English translation wipes out several key clues in the Latin words.)

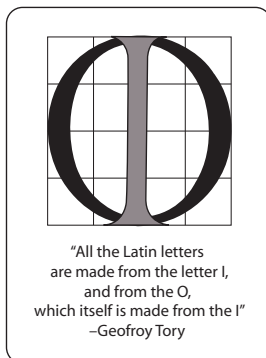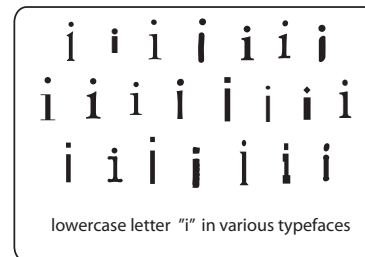Goefroy Tory's 1529 illustration of the "Pythagorean Y" with the Path of Vice and the Path of Virtue

3   On page 5, Dee claims that all Hebrew, Greek and Latin letters were formed from points, lines, and circles. He writes about the IOD, the Hebrew letter "yod", which later became the Greek "Iota," and the Latin "I." He describes an image of a straight vertical line with a circle around it.

+1 line
 2 points
 3 things

(He sees the one straight line and the two points of contact as describing a "Trinity" of things.)

lowercase letter "i"
as a line
with a point at its apex
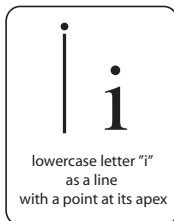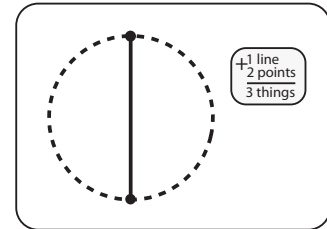
But he really emphasizes the "oneness of the point at the top (apex) of the line. He refers to it as "chireck," which is a dot that acts as a vowel pronunciation mark in Hebrew. He seems to also be referring to a lowercase Latin "i."

lowercase letter "i" in various typefaces

Here is a sampling of "i's" from various typefaces, Aside from the serifs and some fancy styling, they are all essentially a line surmounted by a dot.

"All the Latin letters
are made from the letter I,
and from the O,
which itself is made from the I"
–Geofroy Tory

Geofroy Tory wrote that all the Latin letters "are formed from the I and from the O, which itself is made from the said I."The straight line "I" and the circular "O" provide the proportions and shapes from which all the uppercase Latin letters are formed. Thus, all of the uppercase Latin letters might be formed from various combinations of parts of the Monas symbol. (Tory, pp. 38 – 39)

ABCDEFGHIKLMNOPQRSTVXYZ

All of the Latin letters are made from straight lines and circles.
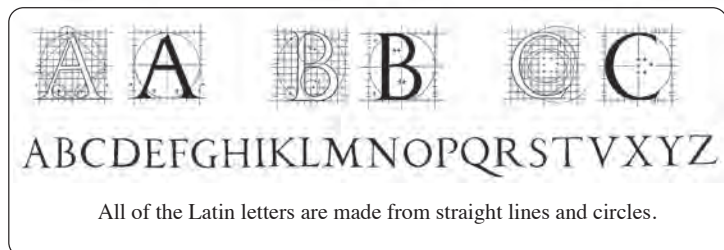
14 professions
(that Dee claims will find the
information in the
Monas Hieroglyphica useful)

1 Grammarians
2 Arithmeticians
3 Geometers
4 Musicians
5 Astronomers
6 Opticians
7 Experts on Weights and Measures
8 Experts on Matter and Space
9 Cabbalists
10 Magicians
11 Physicians
12 Scryers
13 Refiners of Gold
14 Alchemists

4   Dee lists 14 different professions that could benefit from the wisdom of the Monas Hieroglyphica.
Dee's explanations are chock full of clues as to what the Monas is about. But alas, they'll probably sound nonsensical until one understands the 24 Theorems, so we'll return to them later. (pp. 4 – 7 verso).

5   In his advice to ARITHMETICIANS, Dee claims that the "VALUE of the ONE THING which others purport to be Chaos, is primarily explained by the Number **TEN**."

10

6 There is a key clue on the final page of the "Letter to Maximillian." Dee writes that his minded has been "pregnant" with the Monas "**continuously for the past 7 years.**" (His proof is that the Monas symbol appears on the front cover of his 1558 Propadeumata Aphoristica, and is mentioned in Aphorism 52.)

There a lot more to the Letter to Maximillian than this cursory eplanation of a half-dozen clues, but this is enough to get started.

The "Letter to Maximilian" is followed by a "Letter to the Printer, Gulielmo Silvio."

Dee stayed at his friend Gulielmo's house in Antwerp and oversaw the entire preparation and printing process. Not only were Continental printers more skilled than English printers at the time, it was important that the book be printed within King Maximillian's realm.

Dee asks 2 things of Gulielmo: "The first is that you carefully copy (as best you can) the Various Letters, Points, Lines, Diagrams, Shapes, Numbers, and other things."(Dee's list is pretty thorough. What can he mean by other things? (Hint:"intentional errors" and clues in the overall layout of the book are two of these "other things")

7 years

Secondly, Dee asks Gulielmo not to sell the book to just anyone, lest they "torture their minds in incredible ways while neglecting to take care of their everyday affairs."
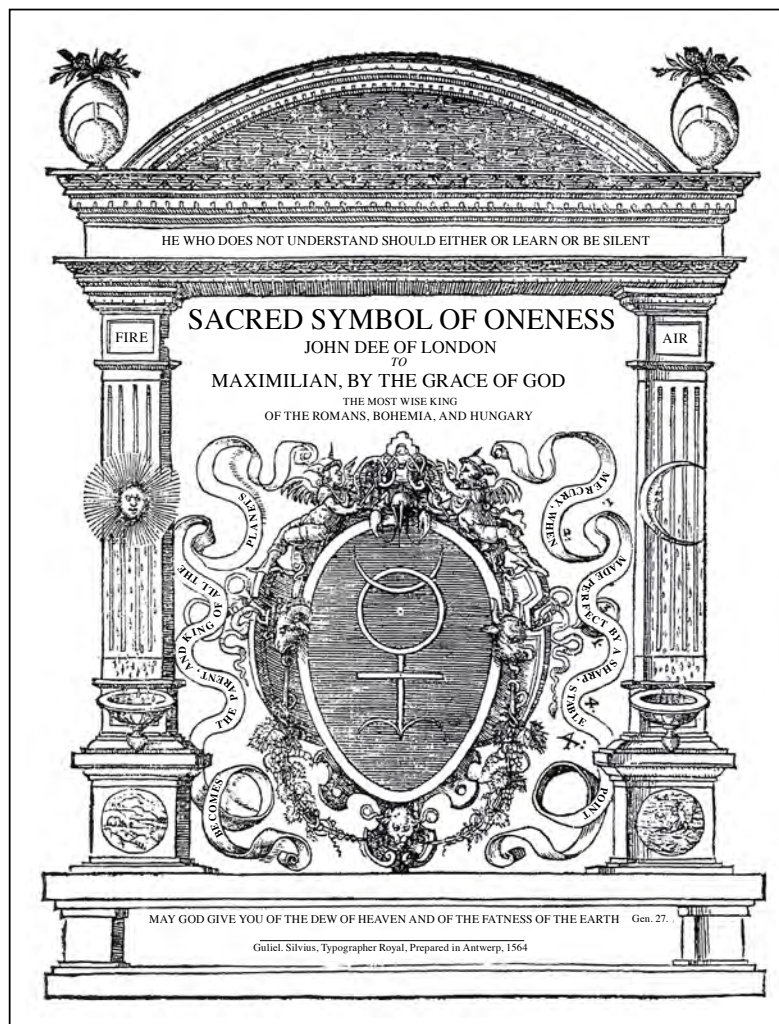
Note that Gulielmo's letter is dated January 30, one day after Maximillian's Letter, dated January 29. The word MONAS which follows the date relates to the title of the book at the top of the very next page. At the bottom of every page you'll see a "carry-over word " that starts the following page (sometimes its a phrase, and sometimes just a syllable). This is a guide for the person on the printer's staff who fold, cuts, and collates the book. (but Dee found it a convenient place to hide a clue).
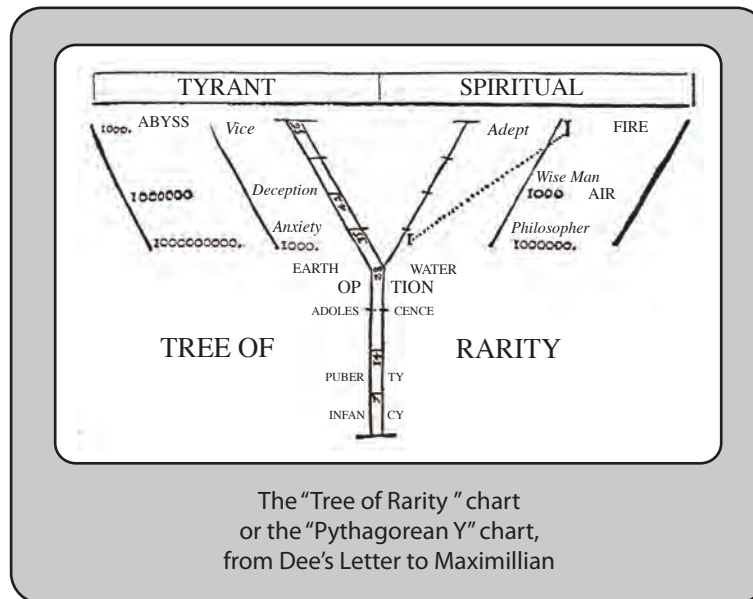
The 24 Theorems

Next, read the 24 Theorems of the Monas. Even if you don't follow what he's trying to say, trudge through it. You may think that Dee is just full of crazy-talk. That's OK. At least you won't think I'm crazy as I explain what it means.

To facilitate our analysis, I've graphically summarized each of the 24 Theorems. In the process, many minor clues have been left behind, but this will help provide a "big picture "of the whole work.

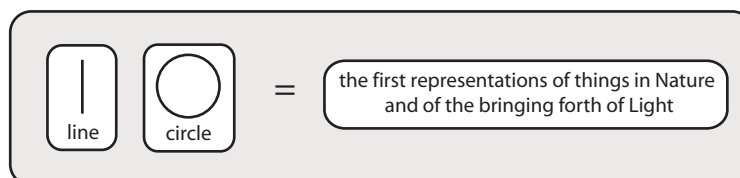# A brief summary and translation of the illustrations from the Monas Hieroglyphica.

HE WHO DOES NOT UNDERSTAND SHOULD EITHER OR LEARN OR BE SILENT

FIRE

AIR

SACRED SYMBOL OF ONENESS
JOHN DEE OF LONDON
TO
MAXIMILIAN, BY THE GRACE OF GOD
THE MOST WISE KING
OF THE ROMANS, BOHEMIA, AND HUNGARY

MERCURY WHEN

MADE PERFECT BY A SHARP

STABLE

POINT

PLANETS

OF ALL THE

AND KING OF

THE PARENT

BECOMES

MAY GOD GIVE YOU OF THE DEW OF HEAVEN AND OF THE FATNESS OF THE EARTH   Gen. 27.

Guliel. Silvius, Typographer Royal, Prepared in Antwerp, 1564

There is only one illustration in Dee's Letter to Maximillian:

The "Tree of Rarity" chart
or the "Pythagorean Y" chart,
from Dee's Letter to Maximillian
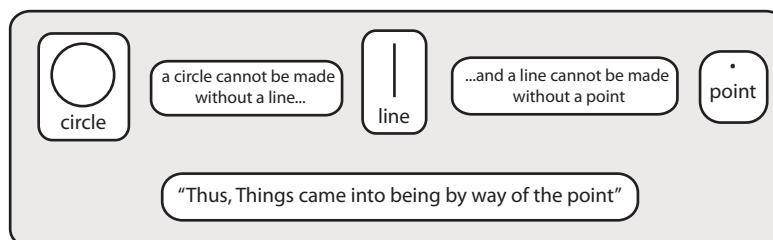
Theorems 1 and 2 sound like the beginning of a geometry textbook:

Theorem 1

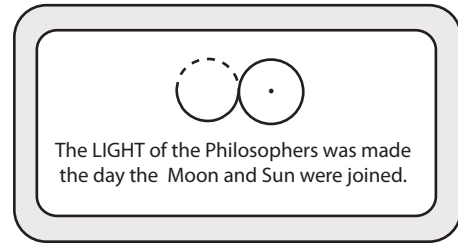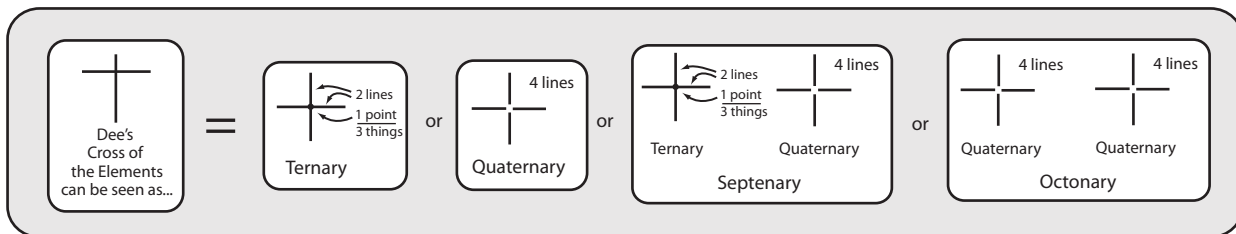line | circle | = | the first representations of things in Nature and of the bringing forth of Light

Theorem 2

circle | a circle cannot be made without a line... | line | ...and a line cannot be made without a point | point

"Thus, Things came into being by way of the point"

Theorem 3

the Sun Circle
has a central point

Theorem 4

the Moon emulates the Sun...

...and during a full Moon,
they appear to be the same size

Theorem 5

The LIGHT of the Philosophers was made
the day the Moon and Sun were joined.

Theorem 6

Dee's
Cross of
the Elements
can be seen as...

$=$

2 lines
1 point
3 things

Ternary

or

4 lines

Quaternary

or

2 lines
1 point
3 things

Ternary

4 lines

Quaternary

Septenary

or

4 lines

Quaternary

4 lines

Quaternary

Octonary

Theorem 7

a line is a flowing of a point...

$=$

...like
a
flow
of
falling
drops

Theorem 8

1+2+3+4=10
Pythagoras'
tetraktys

$=$

Roman Numeral
for 10

$=$

a cross is the
same shape,
simply rotated

3
Ternary

X

7
Septenary

$=$

21
X is the 21st
letter of the
Latin alphabet

Theorem 9

Conjunctio

Sun

Both

Moon

Separatio

A continuous "circular" process
of two things becoming one,
then one becoming two,...

Theorem 10

The Elements
of the Sun and
the Moon
of this Monad,
in which the
Denarian Symmetry
is strong,
want to be separated,
and this is done
with the aid of Fire.

11

Theorem 11

| sign of Aries | = | the first of Aries is the Spring Equinox | = | 12 hours of daylight | 12 hours of darkness | = | [these numbers] "denote our most Secret Symmetries" |

24 hours total

Theorem 12

Saturn + Jupiter + Moon + Lunar Mercury = Lunar Mercury Planets Symbol

These two are "Uterine Brothers"

Theorem 13

Mars + Venus + Sun + Solar Mercury = Solar Mercury Planets Symbol

also in Theorem 13

		4 Lunar Mercury		
		3 Moon		
6 Venus	2 Jupiter		1 Saturn	5 Mars
		Sun		
		7 Solar Mercury		

"Total Inferior Astronomy"

Theorem 14

The Mother — Moon

The Father — Sun

Theorem 15

The Sun is exalted in Taurus

Elements

The Moon is exalted in Aries

12

Dee's
Cross of the Elements
is intentionally "offset"...

...not
equilateral,...

...but both crosses have the same "virtue"
(because the 2 lines are of equal length)

X = Roman numeral for 5 / Roman numeral for 5

Roman numeral for 5 X Roman numeral for 5 = 25 five squared = The Latin letter "V" is the 20th letter and the 5th vowel / 25
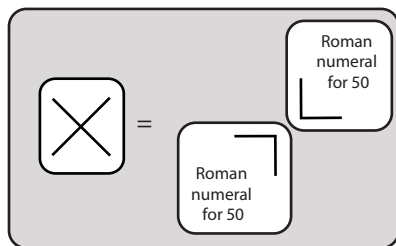
5 is a "circular" number as all of its powers end in 5 (25, 625, etc.)

X = Roman numeral for 50 / Roman numeral for 50

Roman numeral for 50 + Roman numeral for 50 = 100 the "Centurio"

ABCDEFGHIKLMNOPQRSTVXYZ
|——10 letters——|——10 letters——|
The letter L is half-way between
A and X in the Latin alphabet

Roman numeral for 50 X Roman numeral for 50 = 2500

100 the "Centurio" = (Roman numeral for 50 X Roman numeral for 50 = 2500) / (Roman numeral for 5 X Roman numeral for 5 = 25 five squared)

Dee's Cross of the Elements = 1 or 10 or 100

Theorem 17

$\boxed{\times}$ = $\boxed{4 \times \text{Roman numeral for 5}}$ = $\boxed{20}$

$\boxed{+}$ = $\boxed{4 \times \text{Roman numeral for 50}}$ = $\boxed{200}$

Dee's Cross of the Elements

$\boxed{\times}$ = $\boxed{\times \text{ Roman numeral for 10}}$ = $\boxed{10}$

$\boxed{\times}$ = $\boxed{\times \text{ is the 21st letter of the Latin alphabet}}$ = $\boxed{21}$

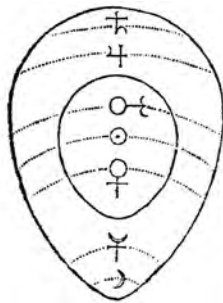$\boxed{+}$ = $\boxed{+ \text{ This is 1 symbol}}$ = $\boxed{1}$

$\boxed{252}$

Dee says there are
"2 other logical ways "
that 252 can be derived
"from our premises"

He wants the reader
about how X separates
into either two L's or two V's
"because then a
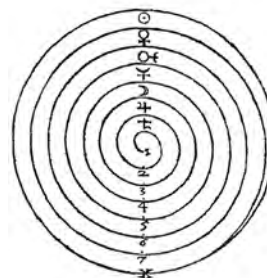LIGHT (LVX) will appear."

Theorem 18

The "EGG" diagram

Based in the information
provided in Theorems 12 and 13,
Dee asks the reader to figure out:

What is the White of the Egg?
What is the Yolk of the Egg?
What is the Shell of the Egg?

He concludes,
"HERE ,
WE HAVE PRACTICALLY THE WHOLE,
IN NATURE'S PROPORTIONS

The "Spiral" diagram

Dee tells the reader that this is the
egg white, egg yolk, and crushed,dissolved egg shells
rolled and rolled in "Spiral Revolutions"
"just as the Scarabs conglomerate their Balls."

Theorem 18

Aesop's Fable of the "**Eagle and the Dung Beetle**"

Cast of Characters:
The hero: A Scarab Beetle who rolls its eggs
in cow dung or horse dung making spherical dungballs.

The villian: An Eagle who lays spherically shaped eggs.

Also starring (with a bit part): A Rabbit

And (playing himself): Jupiter, the chief god of the Greeks,
(synonymous with the chief Roman god Zeus)

A Rabbit, being chased by an Eagle, begged the Scarab Beetle for help.
The Beetle warned the Eagle not to touch the Rabbit,
but the Eagle brushed the Beetle away with the sweep if its wing,
seized the Rabbit in its talons, and devoured it.

Enraged, the Beetle flew up to the Eagle's nest.
Being quite practiced on rolling spherical dungballs,
the Beetle rolled the eggs over the lip of the nest
and they shattered on the rocks below.

When the Eagle returned to its nest,
she was distraught with grief and anger.
The following season, the despairing Eagle implored Jupiter
to provide her with a safe place to keep her eggs.
The great Jupiter allowed her to place the eggs in his lap.

The wily Beetle flew up and deposited
some spherical dungballs among the eggs.
Jupiter noticed the filthy dungballs, was startled, and stood up abruptly.
Once again, all the Eagle's eggs shattered on the ground.

To resolve the whole dispute,
Jupiter commanded that the Eagle lay its eggs in early spring,
when the Beetles are still asleep in the ground.

The moral of the story is that the weak can find clever ways to avenge the powerful, but differences can be resolved.

Following the fable, Dee writes: "*I am not trying to play Aesop, But Oedipus.*"
Oedipus is famous for his riddle of the Sphinx:
"What goes on four legs in the morning, on two legs at noon, and on three legs in the evening"?
The answer: A man, who crawls on all fours as a baby, walks on two legs as an adult, and walks with a cane in old age.
Dee is telling the reader he is using the fable as a **riddle** (Oedipus) not for its **moral message** (Aesop).
He wants the reader to think about how the **spherical eggs and the spherical dungballs
naturally arrange themselves** in Jupiter's lap.

The Greek playwright Aristophanes (ca. 450 BC – ca. 388 BC)
alludes to Aesop's (legendary figure from around 550 BC) fable in his play *Peace***.**
While attending St. John's College in Cambridge (from1542–1545),
Dee was the stage manager for a production of *Peace*.
Using a system of hidden ropes and pulleys, Dee amazed the audience
by having the hero ride a giant Beetle off the stage, disappearing up into the rafters.

Theorem 19

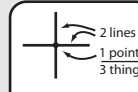The Moon pours out Water (Watery Moisture) and the Sun pours out Fire (Fiery Liquid) into all Earthly Things.

(Water and Fire are opposites, just like Moon and Sun or Mother and Father)

Theorem 20

The differences among these is how that "one point" of intersection is perceived
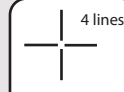
2 lines
Binary

or

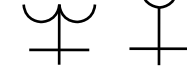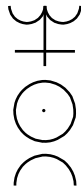2 lines
1 point
3 things
Ternary

or
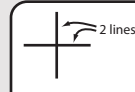
4 lines
Quaternary

Dee concludes,
"Thus we clearly DEMONSTRATE:
THE QUATERNARY RESTS IN THE TERNARY."

Theorem 21

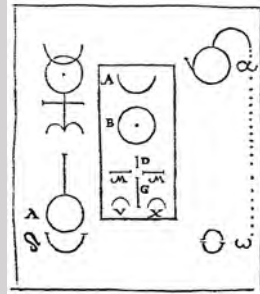Dee separates the inverted Monas symbol into 3 parts

He also shows how the "horns of Aries" might be "closed up" into a circle

Dee draws 3 alternative orientations for the inverted Aries symbol

He suggests that these 3 might also be seen as 6 half circles, "which summed" (3+6) makes "thrice three" (or 9)

Theorem 22
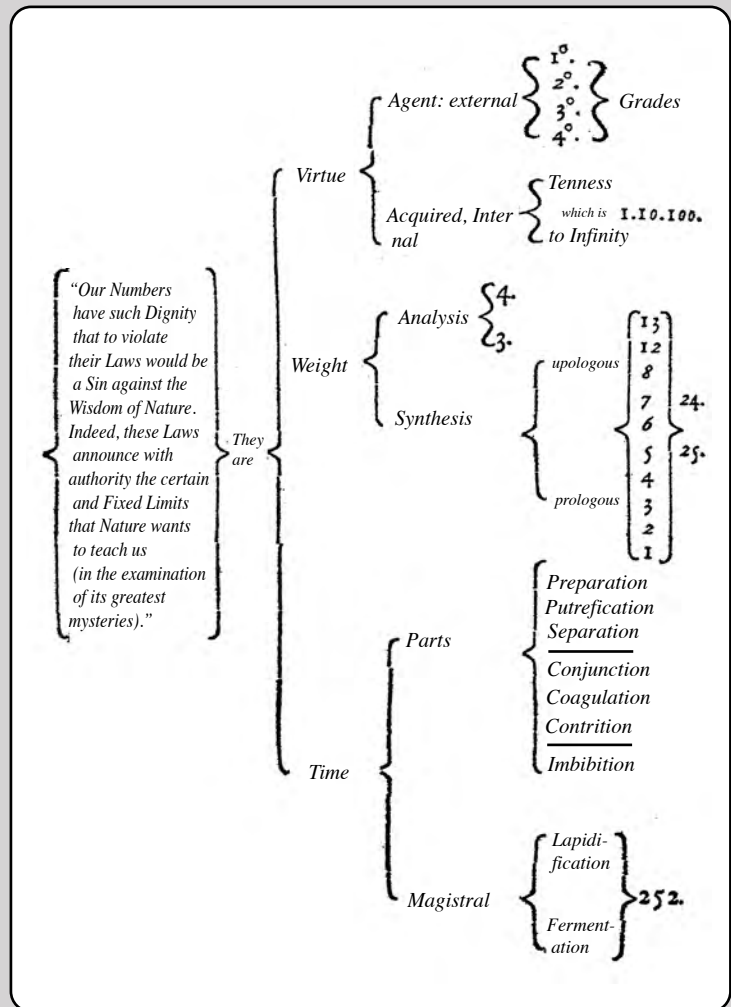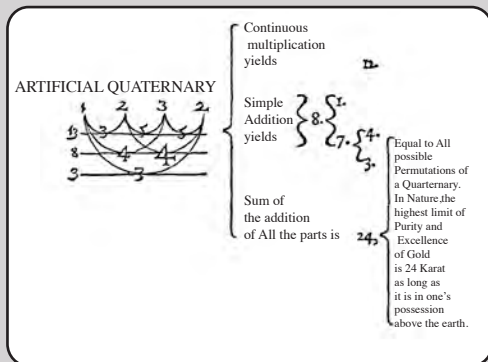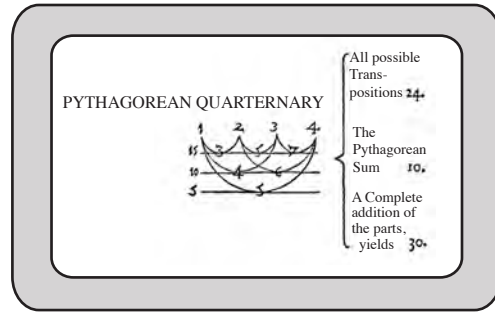

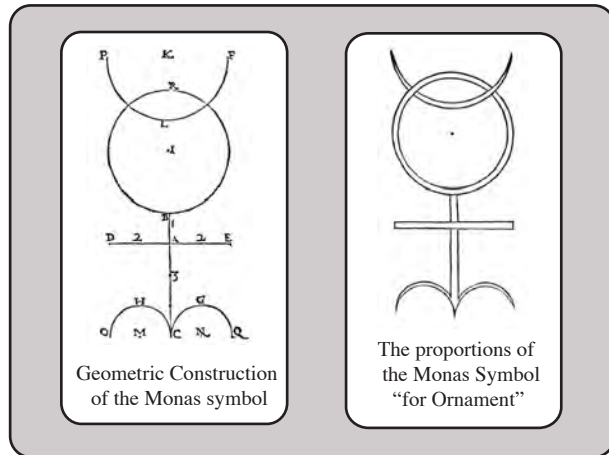Dee calls these the "secret Vessels of the HOLY ART"

Hint: this is actually a visual riddle invoving important numbers

α	Existing before the Elements.	Mortal Adam Male and Female.	Mortification.	Wrapped in Shadow	Born in a Stable
✝	Ordering of the Elements.	Consummation of the Elemental Geneology.	Cross.	Cross.	Sacrificed on the Cross
ω	Existing after the Elements.	ADAM IMMORTALIS.	Vivification	Manifestation	King of Kings Everywhere
Conceived by his Own Influence	Potent Seed	The Creation of Matter.	Earthly Marriage.	Beginning.	α
Suffering and Burial	YHWH Virtue of the Denary	Purification of the Elements.	Martyrdom on the Cross.	Middle	✝
Rising again by his own Virtue	Glorious and Triumphant	Transformationm.	Divine Marriage.	End.	ω

I call this the "36 Boxes" chart

Like the encircled example, all these triads are about a beginning, a middle, and an end.
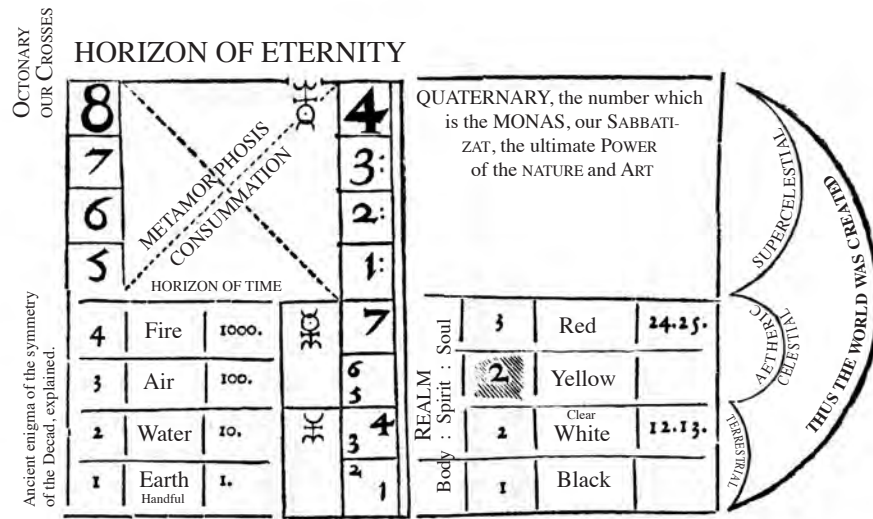
Geometric Construction
of the Monas symbol

The proportions of
the Monas Symbol
"for Ornament"

PYTHAGOREAN QUARTERNARY

All possible
Trans-
positions 24.

The
Pythagorean
Sum 10.

A Complete
addition of
the parts,
yields 30.

ARTIFICIAL QUARTERNARY

Continuous
multiplication
yields

Simple
Addition
yields

Sum of
the addition
of All the parts is 24.5

Equal to All
possible
Permutations of
a Quarternary.
In Nature, the
highest limit of
Purity and
Excellence
of Gold
is 24 Karat
as long as
it is in one's
possession
above the earth.

*"Our Numbers
have such Dignity
that to violate
their Laws would be
a Sin against the
Wisdom of Nature.
Indeed, these Laws
announce with
authority the certain
and Fixed Limits
that Nature wants
to teach us
(in the examination
of its greatest
mysteries)."*

*They
are*

Virtue

Agent: external — 1°. 2°. 3°. 4°. *Grades*

*Acquired, Inter
nal* — *Tenness
which is I.10.100.
to Infinity*

Weight

Analysis — 4. 3.

Synthesis

upologous — 13 12 8 7 6 24. 5 25. 4 3 2 1

prologous

Parts — *Preparation
Putrefication
Separation
Conjunction
Coagulation
Contrition
Imbibition*

Time

Magistral — *Lapidi-
fication
Ferment-
ation* — 252.

The "Artificial Quaternary chart"

Dee writes that "the QUATERNARY, as well as the DENARY impose…certain limits in Numeration."

HORIZON OF ETERNITY

OCTONARY
OUR CROSSES

METAMORPHOSIS
CONSUMMATION

HORIZON OF TIME

Ancient enigma of the symmetry of the Decad, explained.

4	Fire	1000.
3	Air	100.
2	Water	10.
1	Earth Handful	1.

QUATERNARY, the number which is the MONAS, our SABBATI-ZAT, the ultimate POWER of the NATURE and ART

REALM
Body : Spirit : Soul

3	Red	24.25.
2	Yellow	
2	Clear White	12.13.
1	Black	

SUPERCELESTIAL
AETHERIC CELESTIAL
SUBLUNAR
THUS THE WORLD WAS CREATED

"Thus the World Was Created" chart

Theorem 24

All about the number 24
24 hours of the Spring Equinox
1x2x3x4=24
24 in Revelations

Signature

Δ

Dee's signature
Delta, the fourth Greek letter
like D(ee) is the fourth Latin letter

Addendum

INTELLECT
JUDGES
TRUTH

Contact ———— at a Point

The Eye of the Vulgar will, here, be Obscured *and most* Distrustful

*Back cover*

The Watery Dew of Heaven

And of the Fruit of the Earth, He Will Give

What is the "first clue" to unraveling the mystery of these 24 Theorems?

To start this investigation, lets have Mr. Peabody tell Sherman to set the "Way-Back" machine to a Greek colony in Southern Italy, around 550 BC.

The great Pythagoras summarized his mathematical wisdom with his "**tetraktys,**" 10 dots arranged in 4 rows, the way we set up bowling pins nowadays (tetraktys means four-fold).The Pythagoreans felt it was such a powerful description of the cosmos that they made it the basis of their oath.

"By that pure, holy, four lettered name on high,
nature's eternal fountain and supply,
the parent of all souls that living be,
by him, with faith find oath, I swear to thee."

The Pythagorean Tetraktys
- 1 point
- 2 points
- 3 points
- 4 points
- 10 points
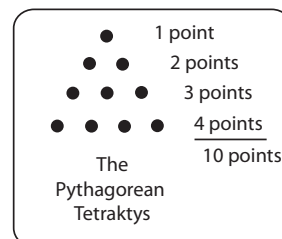
In Theorem 23, Dee mathematically manipulates these 4 digits in three ways:
Multiplication, (or the number of possible permutations of 4 things)
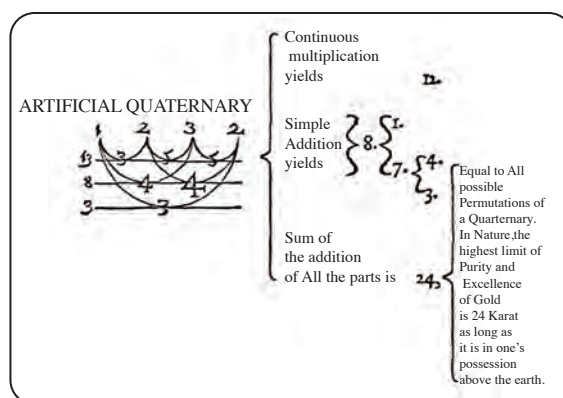$$1 \times 2 \times 3 \times 4 = \mathbf{24}$$
Addition, $1+2+3+4=\mathbf{10}$
A complete addition of the parts, which sum to **30**

PYTHAGOREAN QUARTERNARY

All possible Transpositions **24.**

The Pythagorean Sum **10.**

A Complete addition of the parts, yields **30.**
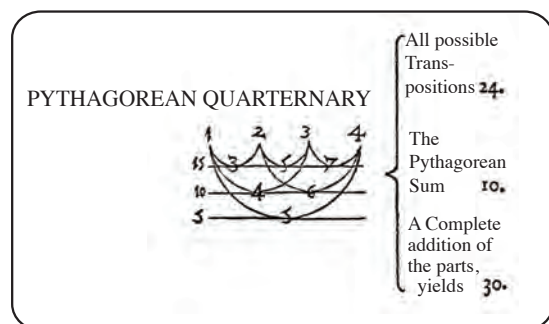
On the facing page, Dee presents his Artificial Quaternary, which is derived from the sequence 1, 2, 3, 2. Why did he change Pythagoras' final "4" into a "2"? (Believe it or not, this minor alteration has huge ramifications that will lead to Dee's rare gift to King Maximillian.)

Don't be misled by the name "Artificial Quaternary." Nowadays we have artificial food coloring, artificial flowers, and artificial Christmas trees, and the word seems to connote an imitation or something fake. But its real meaning is simply "made by man," as opposed to "made by nature." An artificer is one who is skillful or clever in devising ways of making things, like a craftsman or an inventor. (As we shall see, Dee "crafted" this new sequence as a creative way to summarize some of the rhythms that flow through the realm of numbers.)


ARTIFICIAL QUATERNARY

Continuous multiplication yields **12.**

Simple Addition yields **8.**

7. 4.
3.

Sum of the addition of All the parts is **24.**

Equal to All possible Permutations of a Quaternary. In Nature,the highest limit of Purity and Excellence of Gold is 24 Karat as long as it is in one's possession above the earth.

Here are its 3 results:
Multiplication, $1 \times 2 \times 3 \times 2 = \mathbf{12}$
Addition, $1+2+3+2=\mathbf{8}$
A complete sum of all of the parts, **24**

But notice that Dee also does something very strange with the additive result, 8. He inexplicably divides the 8 into 1 and 7. Then he divides the 7 into 4 and 3. What's that all about?
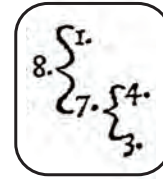

Dee also points out that 24 karat is the highest limit of the purity of gold. Even though (at this point) its unclear why Dee "skillfully devised" the sequence 1, 2, 3, 2, its pretty clear that he has **numbers** on his mind.

Next, we'll turn to Dee's final summary chart, which is labeled "SIC FACTUS EST MUNDUS" or "THUS THE WORLD WAS CREATED."

This is a phrase from the Emerald Tablet claimed to be the work of the Egyptian sage or god Hermes Trismegistus . His last name means "thrice-great," as he was honored by the Egyptians as Thoth, the Greeks as Hermes, and the Romans as Mercury.

The chart appeared to be a hodgepodge of lines, numbers, words, and symbols, but once understood, it is a clear explanation of how number works.
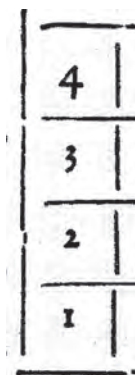
First let's look at the "bottom half" of the chart. It is mostly comprised of quaternaries (groups of four things), though several columns have groups of 2 things or 3 things. The quaternary farthest to the left is the Pythagorean Quaternary "1, 2, 3, 4" (which was our "starting clue).

You'll also see Dee's Artificial Quaternary "1, 2, 3, 2," **except that the order of the sequence has been changed to "1, 2, 2, 3."** The second "2" has hash-marks around it, making it appear as though it was a mistake that was fixed hastily at in the midst of the printing process.

But this seems odd, given that Gulielmo Silvio was such an excellent printer and typographer, and Dee specifically admonished him to "carefully copy" the "Various Letters, Points, Lines, Diagrams, Shapes, Numbers, and other things." (Do you smell a clue?)

Ignis
Aëris
Aquæ
Terræ Pugillus

Next to the Pythagorean Quaternary is the quaternary of the Elements "Fire, Air, Water, and Earth." Under the Latin word for Earth, Dee writes "Pugillus" which means "hand-full" (like a "pugilist" is a boxer who fights with his hands). Dee means the element "Earth" (as in "dirt and stones that you can hold in your hand"), as opposed to the the sphere "Earth" (our whole globe that revolves around the Sun).

The next quaternary is "1, 10, 100, 1000." In Theorem 8, Dee points out that a rotated Cross makes an X, the Roman numeral for 10. In Theorem 16, Dee puts the Cross through various geometric and mathematical contortions to arrive at the "Centurio," the number 100.

In Theorems 16 and 17 Dee says, the "Character of the CROSS is unique, and also represents One." He says "We are now taught… by these Theories of the Cross to enumerate and proceed in this manner: 1, 10, 100. This suggests that the "enumeration" continues to "1000," ""10,000," "100,000," … and on into the large numbers Dee used in the Tree of Rarity chart.

1000.
100.
10.
I.

The number 10 seems to be a key number here, as all the others (except, of course, 1) are multiples of 10. Also, written sideways along the left edge of the chart Dee writes "Ancient enigma of the symmetry of the Decad explained."

What kind of symmetry can 10 have? Maybe halves, with "5" on each side of a see-saw? It doesn't appear to be thirds, as 10 is not evenly divisible by 3. Nor quarters, as 10 isn't evenly divisible by 4 (as 2.5 isn't a whole number). To find out, let's keep an eye on that 10.

The next boxes are not a "quaternary," but only 2 things. The top box has what appears to be the Monas symbol. In the lower box, the symbol that has no Sun Circle is the Lunar Mercury Planets Symbol from Theorem 12.

Logically this means that the "Monas symbol" above it is probably the Solar Mercury Planets Symbol from Theorem 13, rather than the Monas symbol. (Why did Dee make these two symbols identical in the first place?)
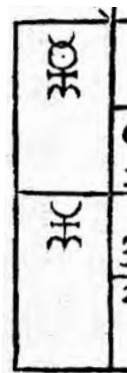
The next quaternary is a strange grouping of the digits 1–7. Notice that "1 and 2" share a box, as do "3 and 4," and also "5 and 6", but "7" is alone. But there is also a "vertical"grouping pattern here. The digits "2, 3, 5, and 6" are in one column and the digits "1, 4 and 7" are in another.

The next group of boxes has a "Ternary" of "Body, Spirit, and Soul" under the heading "REALM." (Hint: a clue has been lost in the translation into English. It is better to study Dee's Latin words "REGNUM, Corporis, Spiritus and Animae."

Here, I''ve rotated it for easier viewing. Curiously, Dee omitted the middle section of the word Spiritus, deleting the letters "irit" and leaving only "Spus"(the accent over the letter "u" is Dee's indication that he has ommited part of the word. He does this frequently in the Latin text). Also, the words are broken up by the grid lines in various ways.

The next quaternary is that curiously "rearranged" Artificial Quaternary, "1, 2, 2, 3."

The next quaternary is a listing of 4 "Stages" in the alchemical process, which are distinguished by various colors. Tenebrae means "darkness or night," and refers to the "nigredo," the beginning of the work where metal or matter is dissolved or putrefied. The alchemists compared this "Black" stage to black things like the crow, the raven, or ebony.
(Lindy Abraham, a Dictionary of Alchemical Imagery, p. 26)

The stage above it is "Chrystallina" or "crystalline," which Dee further labeled as "Serenitas," meaning "clear, fair or serene." This is the "albedo," the "White" stage or "albification." It is sometimes called the white rose, the swan, the dove, lily, snow, alabaster, silver, the moon or the "female principle." (Abraham, p. 215 and 80)

Above that is "Citrinitas," or the "Yellow" stage of the alchemical process. This word derives from the Latin word "citrus," meaning the "citrus" or lemon tree. (In the 1400's and 1500's, this stage was dropped from use by many alchemists, who only counted the other 3 stages.) (Abraham, p. 42)

The final stage is called "Anthrax," which, which nowadays means a disease causing larle boils, or abcesses called "carbuncles" on the skin. But this disease was so-named because those carbuncles looked like "small coals," and the Greek word for coal is "Anthrax."
Coal seems to connote "black," but when heated in a furnace (or to cook burgers) it turns a bright red. This final stage of the alchemical process is called "rubedo" or the Red Stage. It is sometimes called the ruby, the red rose, gold, the sun, or the "male principle."(Abraham, p. 166-169)

These 4 words, "Black, White, Yellow, and Red," summarize the alchemical process, but there is no discussion of these colors or the 4 alchemical stages in the text of the *Monas*. The *Monas* is about primarily about number and mathematics, not alchemy and chemistry.

22

The final vertical column on the bottom of the chart has two blank boxes yearning to be filled. The other boxes have the numbers 12, 13 and 24, 25. Dee has mentioned 12 and 24 in Theorem 10, with reference to the "12 hours of daylight" and the "12 hours of darkness" in the "24 hours of the first day of Aries," adding, "they denote our most Secret Proportions." But why the numbers 13 and 25?

Nowadays many consider 13 to be unlucky, but no one really seems to know why. I personally don't think any number it is unlucky. (In fact, I think 13 is a great sensational number, for reasons that you will soon find out.)

The number 25 is notable for being "5 squared," as Dee mentioned in his mathematical meanderings in Theorem 16.

But the idea that 12 is paired with its neighbor 13, and 24 is paired with its neighbor 25, seems to indicate there is more to these numbers than what I've just described.

To summarize, **six** different kinds of **quaternaries** are listed in the "Below" half of the chart.

Just to the right of the numbers "12, 13" and "24, 25" are two arcing brackets labeled as **Terrestris** (Terrestrial) and **Aethereus Caelestis** (Aetheric Celestial).

Nowadays, Terrestrial means "of the earth," but for centuries it meant anything "sublunar," or "within the orbit of the moon." Though that "round rock of the Moon" seems to be "out there," by observing ocean tides the ancients knew that it had some kind of an effect on Earth.

The word Aetheric in "Aetheric Celestial" comes from the Greek word "aither" meaning "upper air," which derived from "aithen" meaning "burn or shine." So, "Aetheric Celestial" refers to the region of space beyond the Moon where the Sun is located.

The "Aetheric Celestial" arc brackets the Solar Mercury Planets Symbol (of Theorem 13).

The "Terrestrial" arc brackets the Lunar Mercury Planets Symbol (of Theorem 12).

Also within the "Aetheric Celestial" section is the Element of Fire. In the "Terrestrial" section is the Element of Water. As Dee writes in Theorem 19, the Moon pours out Water (Watery Moisture) and Sun pours out Fire (Fiery Liquid) into all Earthly Things.

Also in this section of Lunary Things is the alchemical stage labeled White, the "female principle." This section of Solary Things includes the alchemical stage labeled "Red" or the "male principle."

This corresponds with what Dee wrote in Theorem 14: the Moon is the mother and the Sun is the father. (This is another concept that dates back to Hermes' *Emerald Tablet*.)

23

In short, the section of Lunary Things and the section of Solary Things are one of the most important metaphors in alchemy: the "**union of opposites**" or the "wedding" of male and female.
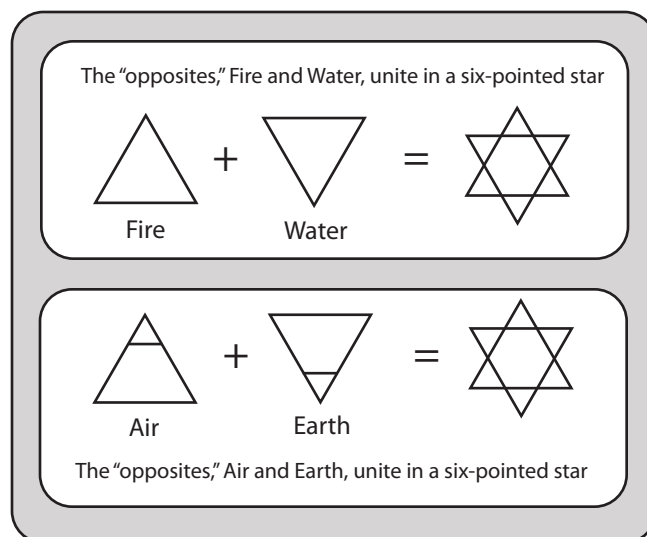

Solary Things									
		4	Ignis	1000.	7		3	Anthrax	24.25.
		3	Aëris	100.	6		2	Citrinitas	
Lunary Things		2	Aquæ	10.	4		2	Serenitas Chryftallina	12.13.
		1	Terræ Pugillus	1.	3 / 2 / 1		1	Tenebræ	

In alchemical metaphor, this "male and female" is frequently represented by a "man and a woman," but sometimes it is extended into the animal kingdom as a "rooster and hen," a "male and female dog," or a "lion and a lioness."

The alchemists graphically depicted the various Elements as a "united opposites."

They illustrated Fire with an "upright" triangle, and Water with an "inverted" triangle. Combined, they unite into a hexagram, the six-pointed star. To modern eyes, this might seem like the Star of David, a symbol of Judaism, but the six-pointed star has been used to symbolize the "union of opposites" by many cultures, for many centuries.

The two remaining Elements, Air and Earth, were considered considered to be opposites as well, as their symbols (triangles with crossbars) also form the same six pointed star.

The "opposites," Fire and Water, unite in a six-pointed star

Fire + Water =

Air + Earth =

The "opposites," Air and Earth, unite in a six-pointed star

Philosophizing about "oppositeness" goes way back in time.

The Chinese sage **Lao Tzu** or "the old one" (604–531 BC), who developed Taoism and wrote "Tao te Ching" (the Book of the Way) saw no such thing things as "good" or "evil." He felt that thinking of "goodness" involved thinking of "non-good-ness." Opposing aspects of life cannot exist in isolation, but must be seen as a whole, as in the yin–yang symbol.

Yin is the female principle associated with the earth, coldness and dark-ness. Yang is the male principle, associated with heaven, warmth, and light. The dark and light parts of the symbol are opposed, but they are interlocked in mutual dependence. The two small spots indicate that each side contains the seed of the other. Together they symbolize wholeness–a perfect circle.
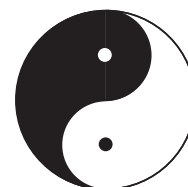
In India, **Siddhartha Gautama** (563–483 BC) who became known as **Buddha**, the "Enlightened One" learned how to attain the sphere of "neither perception nor non-perception." The word "yoga" (meaning union) comes from the Sanskrit root "yuj," which means "to join, connect, or balance."

The Greek philosopher **Heraclitus** (535–475 BC) created a philosophical system that dealt with opposites. "We both step and do not step in the same rivers. We are and we are not." He saw a bow and a lyre as symbols of the "unity of opposites," as they make "backward turning" connections. The curved tension of the bent wood of the bow depends upon the taut string, and vice versa. Of the "journey up the mountain," he saw that the "path up and down is the same one."

In the works of **Plato** (428–348 BC), Socrates and his buddies have prolonged discussions about opposites. The Greeks use the prefix "dia" meaning "through, between, or across" for words suggesting opposites (hence our modern term "diametrically opposed").

    Aristotle (384–322 BC) put his spin on what he knew about the Pythagoreans by providing a table of 10 opposites, (based on the sacred number of the Pythagorean tetraktys).

Aristotle's table of 10 Pythagorean opposites.	
Limited	Unlimited
Odd	Even
Unity	Plurality
Right	Left
Male	Female
At Rest	In Motion
Straight	Curved
Light	Darkness
Good	Evil
Square	Oblong

In the early Renaissance, **Nicholas of Cusa** (1401–1464) wrote a treatise with the paradoxical title "On Learned Ignorance" in which he sees God as a "**coincidentia oppositorum,**" a "coincidence" of opposites.

"In God we must not conceive of distinction and indistinction,
for example as two contradictories,
but we must conceive of them
as antecedently existing in their own most simple beginning,
where distinction is not the other indistinction."

(Nicholas of Cusa, Selected Spiritual Writings, H. Lawrence Bond, New York, Paulist Press, p.29)

After Dee's time, many authors have written about "opposites," including:
Immanuel Kant (1724–1804)
Georg Hegel (1770–1831)
Neils Bohr (1885–1962)
Jiddu Krishnamurti (1895–1986) (who influenced David Bohm)
Claude Levi-Straus (1908–present)
David Bohm (1917–1992)

The alchemists also used the ancient symbol an "**ouroboros**," a serpent swallowing its tail, as a symbol for the union of opposites or the cyclical nature of the universe. This symbol dates back to the Greeks, the Phoenicians, and even the ancient Egyptians. The Roman author **Horappolo** wrote a text called "*Hieroglyphica*" (sound familiar?) about the Egyptians in which he says: "When they wish to depict the Universe, they draw a serpent devouring its own tail…" (Horappolo, in Boas, p.43)

In 1556, the Italian author Giovanni Piero Valeriano Bolzani wrote a commentary on Horappolo's *Hieroglyphica* (which Dee had in his library). (Roberts and Watson, book number 114)

George Boas published a modern edition of *Hieroglyphica* in 1950. Boas found references to the Egyptians use of this symbol in several Roman authors including Honoratus, Claudian, and Macrobius (authors whose works Dee also had in his library).

When the Greeks ruled Alexandria, Egypt around 150 AD, a female alchemist named **Cleopatra** (not the Queen) illustrated an ouroboros in her *Chrysopeia* (The Making of Gold). At the center of the illustration are the Greek words "hen to pan" which means "one, the all" or "All is one." The top half of the serpent is jet black, and the bottom is light-colored with a spotted texture.

An ouroboros also appears in Andrea Alcati's (1492–1550) *A Book of Emblems* (which Dee had acquired in 1543). (new edition by John F. Moffit, McFarland, 2004) (Roberts and Watson, book number 823,)

In the 1900's, the Swiss psychologist **Carl Jung** (1875 – 1961) described the ouroboros as alchemy's most basic form of a mandala, "the ouroboros is a dramatic symbol for the integration and assimilation of the opposite, i.e. of the shadow…. He symbolizes the One, who proceeds from the clash of opposites, and he therefore constitutes the secret of the prima materia… (Carl Jung, Collected Works, volume 14, paragraph 513)

### *"Oppositeness" in the parts of the Monas Symbol and in words of the Monas text.*

"The Sun and the Moon" is Dee's main way of saying "union of opposites". In Theorem 4, Dee writes that the "Moon emulates the sun" and during the full Moon, they appear to be the same size.
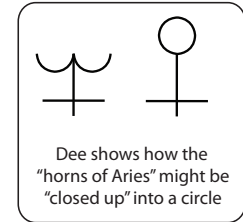
the Moon emulates the Sun…

…and during a full Moon, they appear to be the same size
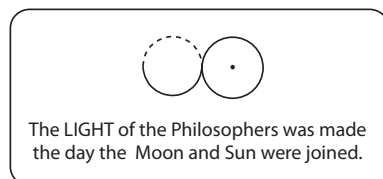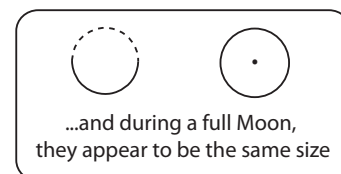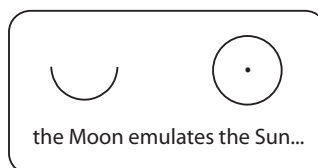
In Theorem 5, Dee tells us that the day of the Moon and Sun were joined, "the LIGHT of the Philosophers was made."

The LIGHT of the Philosophers was made the day the Moon and Sun were joined.

I've portrayed them as two circles touching, but the two circles should perhaps should be shown as I've done in my graphic depiction of Theorem 9, when two things become one.

Conjunctio

Sun

Both

Moon

Separatio

26

Two identical sides
of the Aries symbol
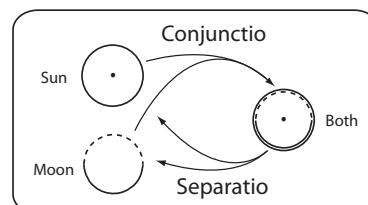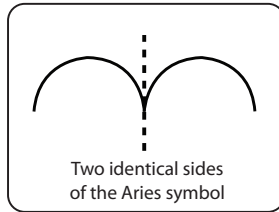
One can even sense the idea of the "union of opposites" in the symmetrical Aries symbol. Two half circles meet at a point.

In Theorem 21, Dee shows the two "horns" of the inverted symbols closing up into a circle.

Dee shows how the "horns of Aries" might be "closed up" into a circle

And it's not hard to see the idea of "union of opposites" in the Cross of the Elements. Though Dee depicts it as an "off-set" cross in the Monas symbol, its clear from his text that it is equivalent to an equilateral cross or and equilateral X-shape.

Roman numeral for 5

Roman numeral for 5
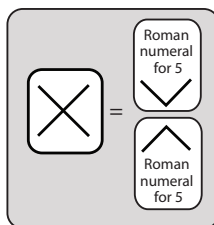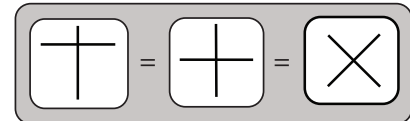
In Theorem 20, he calls the Cross "Binary" (two lines) and goes into lengthy explanation of the intersection point. In Theorem 11, Dee he portrays the X as a union of two opposing V's. He also shows the X as the union of two opposing L's.

Roman numeral for 50

Roman numeral for 50
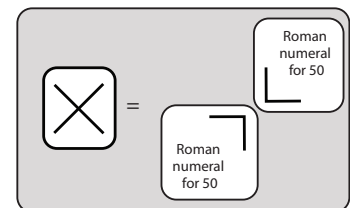
Interestingly, a character in John Brigg's article, "*Alice in the Looking Glass of Art*" proclaims,
"The World is a Big Reflecting X. That's clear."
Another character adds,
**"But the Big Reflecting X is everywhere if you look.
Like William Blake said, isn't the Universe in a grain of sand?"**
To which the first replies,
"Or the blackbird's eye against twenty snowy mountains."
(published in Lewis Caroll's Lost Quantum Diaries, Williams Shanley editor, Germany, Werner Locher)

This metaphor (or "reflectaphor," as Briggs likes to call it) is a reference to the first 3 lines of a classic poem by the American poet Wallace Stevens (1879-1955) that paints a word-picture of oppositeness:

Among twenty snowy mountains
The only moving thing
Was the eye of a blackbird.

(Wallace Stevens, "Thirteen Ways of Looking at a Blackbird")

Implicit in this sentence are all these opposites:

black-white
small-large
one-many
moving-still
living-nonliving
round-angled

Metaphors, similes, and puns all juxtapose things that reflect each other, and Elizabethan loved this literary fun. In his letter to Maximillian, Dee even admits that his using "Gematriam" (letters representing numbers) "Notariacon" (letters represent concepts" and "Tzyruph" (jumbled letter or word codes.) In short, the idea of "oppositeness" is a huge theme in the Monas.

It clear Dee was fond of puns. He once in the marginalia of a text "my REDY friend." referring to his friend Sir Edward Dyer. He also loved to coin new words, like Arioton (Aries + ton) and Acioades (Acies, sharp + aedes) and his new name for geometry, megethological (magnitudes + the study of). (General and Rare Memorials, p. 80) (Jostin p. 137 and 161, and Dee's Preface to Euclid, p. aiiij. and aij verso.)

In summary, the idea of opposition is a huge theme in the Monas. This raises the question, if the Monas is about number, how does "union of opposites" apply to number?

Opposites on the Title Page
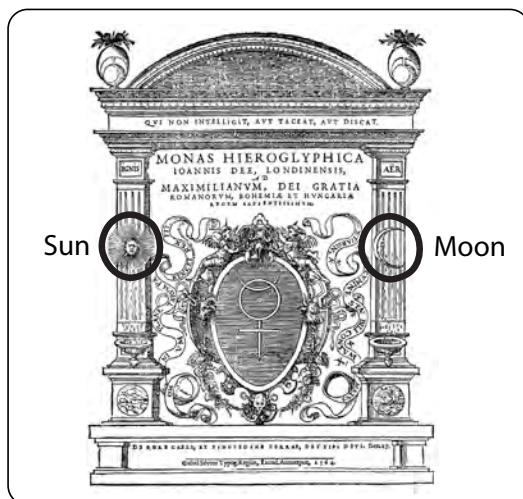
The ideas of of the union of opposites in the "Thus the World Was Created" chart are echoed on the Title Page. Here we see the Sun prominently displayed on one of the columns and the Moon on the other.

The Sun is pouring out its "Fiery Liquid" and the Moon is pouring out its "Watery Moisture."
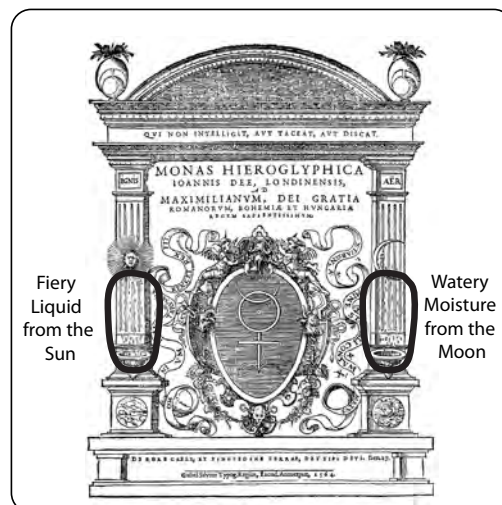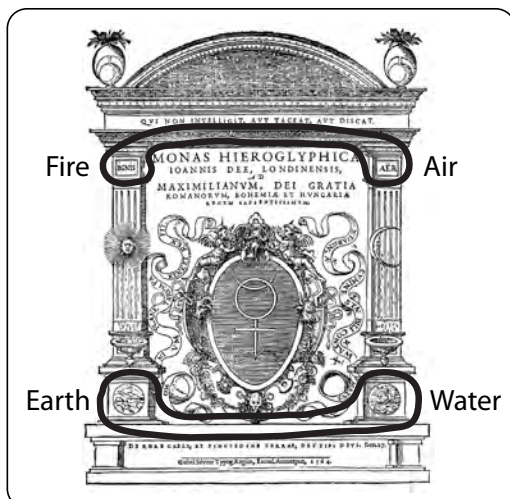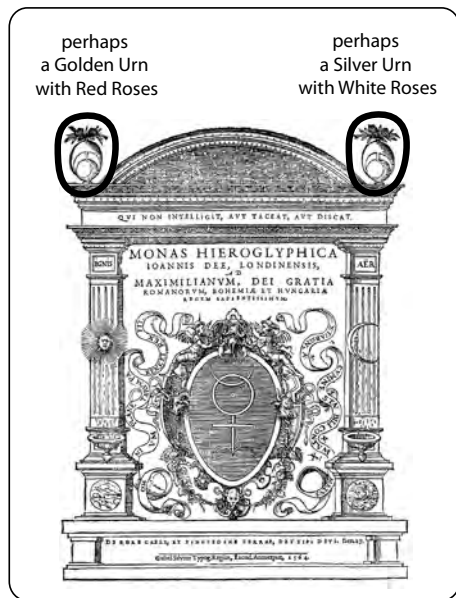

At the top of the column are the words Fire and Air. On the pedestals are circular illustrations depicting Earth (left) and Water (right).

perhaps a Golden Urn with Red Roses

perhaps a Silver Urn with White Roses

Though the Title Page was printed in black-and-white, we might even imagine the upper left urn as a Golden urn holding Red roses (the flower buds are hard to see among the leaves, but they're visible). The upper right urn might be perceived as a Silver urn holding White roses.

Regarding the "whole" architectural structure on the Title Page, the **two columns** are a sturdy pair of opposites, united by the upper entablature and dome. The inner edge of the left column in shadow and the inner edge of the right column is illuminated, hinting at another pair of opposites, **dark and light**. The dome (Above) might also be seen as the opposite of the foundation (Below).

In the central embem there are **2 Mercuries, 2 horned animals, 2 cascading grapevines, and 2 "flowing ribbons."**

Creativity and "Oppositeness"

The opening paragraph of **John Briggs**' *Fire in the Crucible* (the liner notes call the book the "Alchemy of Creative Genius") paints a picture of a "medieval alchemist in a flickering chamber surrounded by bubbling retorts", with beakers with "strange colored fluids and precipitating crystals." The alchemist is focused on the alembic or "philosopher's egg," intent on fabricating the "prima material, the First Matter."

Richard Cavendish (author of *Man, Myth, and Magic*) explains "First Matter is not matter in the normal sense of the term, but the possibility of matter." Briggs adds, "The First Matter for a creator is the distillation of vision."

Modern day creative people "work like the ancient alchemist to distill, then further distill that vision until it condenses, coalesces and emerges in a miraculous form." Briggs explains that creators and alchemists "crave possession of a material or *object*-ive form (a scientific law, a painting, a concerto – a philosopher's stone) to capture something exquisitely nonmaterial like a truth or a subtle perception." (Briggs, pp. 3 and19)

He adds that "creators create in order to find some truth about life and we value them precisely because we see that they have found it and the bequeathed to us their mind-altering vision." (Briggs p. 9)

OMNIVALENCE

Briggs uses the term "omnivalence" to describe the state of combining opposites to see "possibilities, potentials, mystery, openness." ("omni" means "all," valence means "strength")

Omnivalence wraps its arms around both polarities so they aren't seen as competitors, but as cohorts in a larger scheme of things. Briggs gives the example that perfume is often made of "such contrary substances as skunk oil and flowers." (Briggs, p. 112)

Briggs gives Expressionist sculptor **Louise Nevelson** as another example. In various parts of her autobiography, calls herself an "architect of shadow," and an "architect of light."She even calls herself "architect of reflection." (Briggs p. 183)
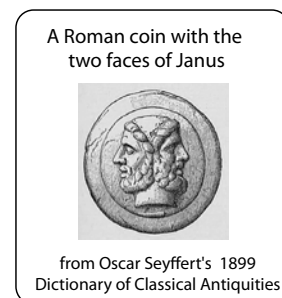
Briggs cites **Albert Rothenberg**'s comment that Mona Lisa's smile has been described as "both good and wicked," as well as both "compassionate and cruel." There is something "more" in this tender balance that conveys to the viewer "some elusive and universal truth." The contraries somehow mold together, like the way the author Virginia Wolf said she was able to "achieve a symmetry by means of infinite discords." (Briggs, p. 113)

The physicist **David Bohm** saw omnivalence as the "space between" opposites, saying, "If you say north and south, then there's a whole range in between. Therefore you have to tremendously enrich the field to a new level in order to resolve the opposites." He continues, "if you hold these opposites together, then you must suspend thought and your mind must move to a new level. The suspension of thought allows intelligence beyond thought to act. Then you can create a new form." (Briggs' interview with Bohm, in Briggs, p.114-115)

Rothenberg points out a subtle distinction in how creative people view the opposites in omnivalence. They don't "flip" from one opposite to another, like a ball in a tennis match. Nor do they combine the two opposites to "reconcile" them or make a "synthesis." Instead, they have the ability to view the oppositions "simultaneously." They can accept that the paradox is unresolvable, yet still express both sides in the same breath. (Briggs, p. 184)

Rothernberg calls this "janusian" thinking after the Roman god Janus who had two faces. He was the "God of the Gate," as his two faces could observe all comings and goings. (He was frequently carved on gateposts and doorways, from which we get our word for the caretakers of the doors and halls: janitor.)

Janus as also the god of "endings and beginnings" and thus the start of the new year, January, was named after him.

A Roman coin with the two faces of Janus

from Oscar Seyffert's 1899 Dictionary of Classical Antiquities

Creative consultant Tom Monahan recommends "Intergalactic Thinking," applying ideas and principles from one field in another field. To spark creativity Monahan also recommends "180 degree thinking: directing your thought process in the exact opposite direction of where conventional wisdom would suggest to go. If conventional wisdom says think 'soft', think 'hard'. If it says 'warm,' think 'cold.'" He calls it a "tnereffid" way to ideate. (tnereffid is "different" sdrawkcab)

(Tom Monahan, *The Do-It-Yourself Lobotomy, Open your Mind to Greater Creative Thinking*, pp. 98,108)

John Dee was a creative thinker. He was fascinated by the idea of opposites and saw how the study of opposites could be applied to many foelds from alchemy to language to optics to architecture, and even mathematics. Aphorism 9 in his *Propadeumata Aphoristica* reads,

"Whatever is in the Universe
possesses order, agreement and similar form
with something else."

The "Above" half of the "Thus the World Was Created" chart

Dee labels the arc that brackets the "top half" of the chart SUPERCELESTIAL. This is the region beyond the TERRESTRIAL region of the "Lunar Planets," and beyond the AETHERIC CELESTIAL region of the "Solar Planets." It's way out there in the realm of the fixed stars or what might be called "heaven," the abode of God (or gods) and the angels.

I simply call it the "Above" part of the chart, as opposed to those 2 sections of the "Below" part of the chart. According to the Hermetic axiom "As above, so below," we should expect to find some similarities. But they are not very obvious.

The most prominent features of the "Above" part of the chart are the digits "1, 2, 3, **4**" and "5, 6, 7, **8**." Note that they are grouped as two quaternaries and the numbers 4 and 8 much bolder than the other digits.

Running vertically near the digit 8 are the words "OCTONARIUS nostrae CRUCES," meaning "The OCTONARY, our CROSSES."

Dee calls the cross "Octonary" in the Theorem 6, explaining, "Each line might, for this purpose, be twice repeated." He seems to be saying that 2 crosses, each comprised of 4 lines, have 8 lines in total.
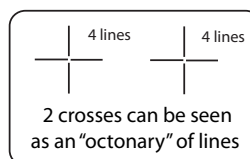
This idea is echoed in the "36 Boxes" chart of Theorem 22, in which two neighboring boxes have the word "Crux" (Cross) in them.

Why does Dee arrange these 8 digits as two quaternaries this way?
Why are the 8 and 4 bold?
And why do the digits 1, 2 and 3 have colons next to them?
(Stay tuned for the answers.)

Along the large, "dashed-line X" are the words METAMORPHOSIS and CONSUMMATA. Dee uses these words in Theorem 24, "we shall consummate and conclude with the METAMORPHOSIS of all the Transpositions of the Quaternary, defined by the number 24."

31

He calls the "Spiral Diagram" of Theorem 8 the "METAMORPHOSIS OF THE EGG."
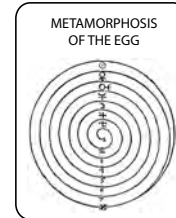
He also uses the word CONSUMMATUS ("meaning "to make perfect") in the "maxim on the flowing ribbons" of the Title Page. What the heck can they mean? Metamorphosis is the process of transformation, like the caterpillar to butterfly. What does "trans-formation" have to do with all Dee's favorite numbers?

Consummata means "to make perfect, to finalize, to complete a transaction" (or a mar-riage). It's a combination of the Latin prefix "con-" meaning" altogether" and summa" meaning " sum total."

The "maxim of the flowing ribbons" on the Title Page reads:
"Mercury, when made perfect by a sharp, stable point
becomes the parent, and King of all the planets."
How does a sharp tip consummate a planet so it becomes a parent or King to other planets?
This maxim sounds like astrology-speak, but (hint) it really has to do with numbers.

Let's continue the tour of Dee's summary chart. One line of the "giant dashed-line X" intersects an inverted Monas symbol, hanging upside down like a bat. Note that the perimeter line of the chart is intentionally broken just above it. Another clue is that it is aligned almost directly above the Solar Mercury Planets Symbol and the Lunar Mercury Planet Symbol.

The upper-right quadrant of the chart reads,
"QUATERNARY, the number which is
the MONAS, our SABBATI-
ZAT, the ultimate POWER
OF NATURE'S ART."
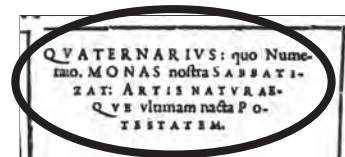
That sounds pretty strange. And what the heck is a SABBATIZAT? It sounds like the "Sabbath" – the seventh and final day of the week, the day of rest (from Friday evening to Satur-day evening for the Jews and Sunday for most Christians.) A "sabbatical" is a university profes-sor's period of paid leave, which is generally every seventh year. In Biblical times, a "Sabbath" also meant every seventh year, when the land was allowed to remain fallow.

Dee seems to be referring to the number seven here. If so, this statement references three of the single-digits, 1, 4, and 7 (Monas, Quaternarius and Sabbatizat).

These are the same three digits that Dee highlights in the "Below" half of the chart by putting them in their own separate column. What's so special about 1, 4 and 7?

And what does Dee mean by the "ultimate power of Nature's Art"?

If it's a number, its most likely the number **10**, the base number of our Base Ten system. Ten, the first "double-digit" number seems to rule over the "single digits" 1, 2, 3, 4, 5, 6, 7, 8, and 9.

Dee owned a copy of **Henry Cornelius Agrippa's** (ca. 1486-1535) *Three Books of Occult Philosophy*, in which Agrippa writes:

"The number ten
is called every number, or a universal number, complete,
signifying the full course of life:
for beyond that we cannot number, but by replication."
(Roberts and Watson, books 742)
(Agrippa, in Tyson, p. 287)

Dee is pretty enthusiastic about the number 10 in his advice to Arithmeticians in his Letter to Maximillian:

"Will he not be filled with the greatest admiration
by this most subtle, yet General Evaluating rule:
that the strength and intrinsic VALUE of the ONE THING,
purported by others to be Chaos,
is primarily explained
(beyond any arithmetical doubt)
by the number TEN?"
(Dee, Monas, p. 5 verso)

If Dee's sentence in the upper-right quadrant is inferring "**1, 4, 7,** and **10**," why does he only emphasize "**1**, **4**, and **7**" in the listings in the "Below" half of the chart? **Where's the 10?**

Well, we don't have to look very far to find a **10**. Not only is there a 10 in the "1, 10, 100, 1000" quaternary, but there's also a 10x10, and even a 10x10x10.

Further to the left is the Pythagorean Quaternary 1, 2, 3, 4 that sums to 10, that was discussed in Theorem 8: "As Pythagoras himself used to say, 1, 2, 3, and 4 add up to ten." Dee illustrates this again in the Pythagorean Quaternary of Theorem 23.

Dee also refers to **10** in the cryptic assertion,"Ancient enigma of the Symmetry of the Decad, explained."

Even though "10 divided by 3" does not make a whole number, "**1, 4, 7,** and **10**" do have a certain harmony in the sense that they are each **"3"** apart from each other.
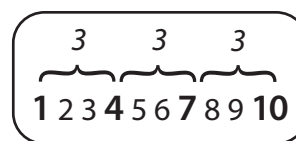
### *Suddenly it becomes clear what Dee's referring to: his Monas symbol!*

If you study Dee's geometric construction of the Monas symbol in Thereom 23, you'll see that its "spine, (which he calls line CK)" is made of 10 points. After a detailed explanation, Dee adds:

> "We should point out to the Mechanicum
> that the whole here consists of nine equal parts,
> each the length of out Fundamental AB.
> Thus he may go about performing his work in another way.

(In the Preface to Euclid, Dee describes a "Mechanicum" or "mechanician" in English, as a "Mechanical workman who utilizes a mathematical principle without necessarily understanding its derivation.)
(Dee, Monas, p. 24, verso, and Dee, Preface, p. aiij verso)
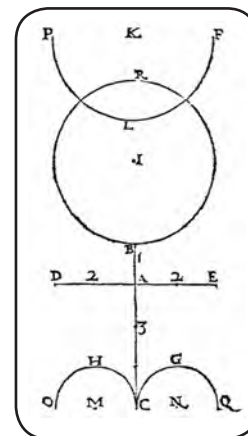
Dee is suggesting that the Monas symbol can be seen as either "10 points" **OR** "9 parts."
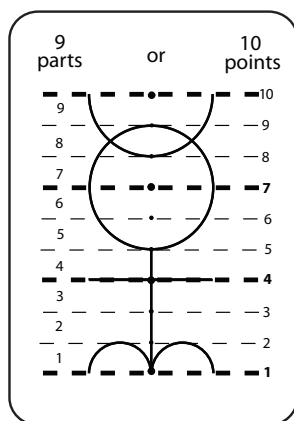
The thing that confirms that he saw this arrangement as the Symmetry of the Decad is that point 1, point 4, point 7 and point 10 are all located at the **centerpoints** of the 4 parts of the Monas Symbol.

Point 1: The middle tip of the Aries sign
Point 4: The intersection point of the Cross of the Elements
     (Here's a good reason why Dee used an "offset" cross)

Point 7: The centerpoint of the Sun Circle
Point 10: The centerpoint of the Moon half circle

We're making progress, and this arrangement is very nice, but it's not really an "explanation" of any "Ancient enigma." There must be more to it than this.

The only printed features of the chart we haven't covered are the large words HORIZON AETERNITATIS (Horizon of Eternity) and HORIZON TEMPORIS (Horizon of Time or the Temporal Horizon).

Michael Schneider has written an insightful text on the qualities (as opposed to the quantities) of the single digits *"A Beginning Guide to Constructing the Universe, the Mathematical Archetypes of Nature Art and Science,"* in which he calls the chapter on the "Number 9" simply "The Horizon."

He writes the "ancient Greeks called nine '**the horizon**,' as it lies at the edge of the shore before the boundless ocean of numbers that repeat in endless cycles the principles of the first nine digits." (Schneider, p. 302)

As Dee's chart seems to be all about numbers, it makes sense that number 9 is sitting (disguised) on top of the number 8. It makes you wonder, "Where is the 10"? Well, you don't have to look very far, because that large dashed-line X can certainly be seen as a 10, as Dee propounds in Theorem 8 and in Theorem 16.
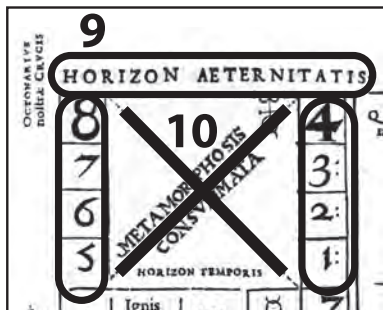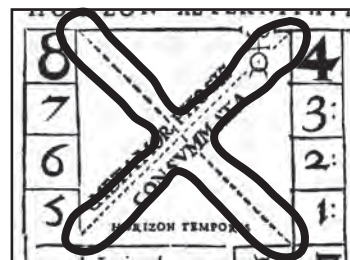
Now we have a full set of bowling pins in the heavens; Pythagoras would be pleased.

If Dee is following the Hermetic axiom "As above, so below," we should also expect to find 1, 2, 3, 4, 5, 6, 7, 8, 9, and 10 in the "**Below**" half of the chart.

The smaller capitalzed words "HORIZON TEMPORIS" ("HORIZON of TIME") are actually in the "Above" part of the chart, but seem to refer to the top line of the "Below" part of the chart. You can see that it would have graphically messed up a few quaternaries to place it in the "Below" half.

However, even if this is seen as contributing a "9" to the Below half, there is still no "8" in the Below half.

I took a closer look at the organization of the "Below" half of the chart. The Lunar Mercury Planets Symbol seemed associated with 1, 2, 3 and 4. The Solar Mercury Planets Symbol seemed associated with 5, 6, and 7. This correlated with the arrangements Dee discussed in Theorems 12 and 13. Replacing 1, 2, 3, 4, 5, 6, and 7 with the appropriate planets looks like this:

	7	Sun, and also
	7	**Solar Mercury**
	6	Venus
	5	Mars
	4	**Lunar Mercury**
	3	Moon
	2	Jupiter
	1	Saturn
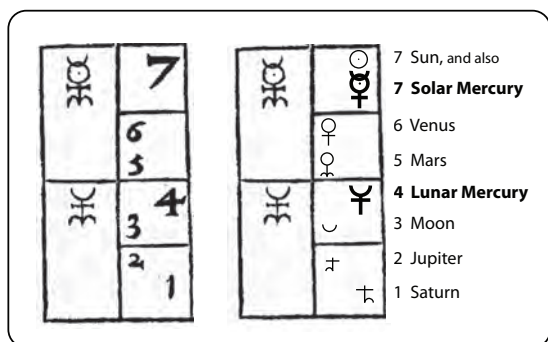
In Theorem 13, Dee called Solar Mercury the "Uterine brother" of Lunar Mercury. As Lunar Mercury is 4 and the Solar Mercury is 7, this seems to agree with why Dee wrote the digits 4 and 7 bolder than the rest.

It also helps explain why there are no planets labeled "8" in the "Total Inferior Astronomy Diagram" of Theorem 14.

Theorem 13

"Total Inferior Astronomy"
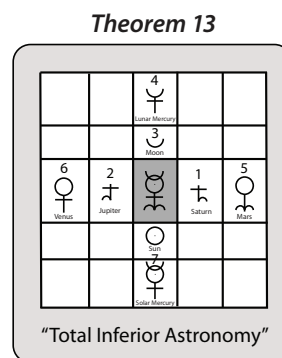
You've probably already guessed probably guess where Dee "hid" 8 and 9.

**The Solar Mercury planets symbol is 9
and
the Lunar Mercury Planets symbol is 8**

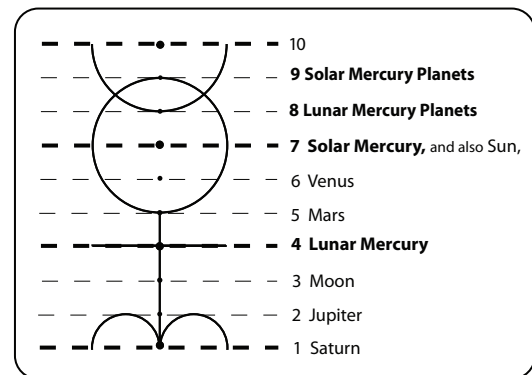| 9 Solar Mercury Planets		7 Sun, and also
		7 Solar Mercury
		6 Venus
		5 Mars
8 Lunar Mercury Planets		**4 Lunar Mercury**
		3 Moon
		2 Jupiter
		1 Saturn

This revelation sheds light on what the Yolk White and Shell are in the "EGG" diagram of Theorem 18.
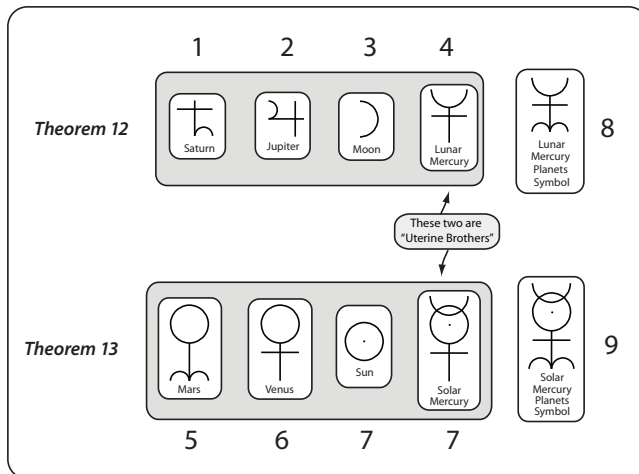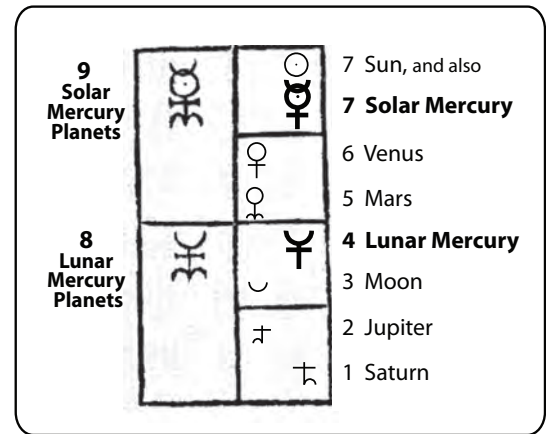
Dee introduces Theorem 18 by referencing Theorems 12 and 13. "From our twelfth and thirteenth theorems it may be gathered that celestial astronomy is alike a **parent and master** to the Inferior [Astronomy]. [emphasis mine]

Next, he says, "we shall behold an Anatomy exactly corresponding to that of our Monas…"

This is what we just saw previously: Lunar Mercury (4) aligned with the centerpoint of the Cross (point 4) and Solar Mercury (7) aligned with the centerpoint of the Sun circle (point 7). We've also found 8 and 9, but 10 is still unaccounted for.

	10
	9 Solar Mercury Planets
	8 Lunar Mercury Planets
	7 Solar Mercury, and also Sun,
	6 Venus
	5 Mars
	4 Lunar Mercury
	3 Moon
	2 Jupiter
	1 Saturn

He then adds, "As we were contemplating both the Theoretical and Heavenly motions of that celestial **MESSENGER [Mercury]** we were taught that the figure of an EGG might be applied to these coordinations." (again, emphasis mine)

Have you cracked the case?
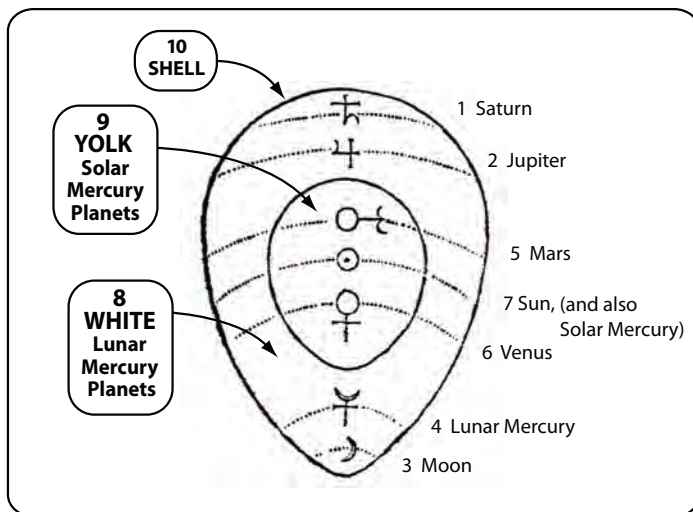
The **White** of the Egg
represents the number **8**,
the Lunar Mercury Planets,
which are
Saturn (1), Jupiter (2),
Moon (3) and Lunar Mercury (4).

The **Yolk** of the Egg
represents the number **9**,
the Solar Mercury Planets
which are
Mars (5), Venus (6), and Sun (7).

And the **Shell** of the Egg,
which encompasses them all,
is the number **10.**

I have emphasized two clues in the quotes I just presented from Theorem 18. The "celestial MESSENGER" is a reference to **Mercury**, the fleet-footed messenger of the Roman gods (with wings on his sneakers). Dee also says Celestial Astronomy is alike a "**parent and master**" to Inferior Astronomy.

An alert reader would recall that Dee refers to both of these concepts in the "maxim of the flowing ribbons" on the Title Page:

"**Mercury**, when made perfect
by a sharp, stable point
becomes the **parent, and King**
of all the planets."

While wondering what Dee meant by "parent, and king of all the planets," I noticed one very small detail. (Remember Dee earnestly asked the printer Guilielmo Silvio that he "carefully copy…the Various Letters, Points, Lines, Diagrams, Shapes, Numbers, and other things" so it would "perfect in every part.")

There was a **comma** between the word "parent" and the words "and King." The word parent seemed to be referring to **something different** than the King.

We've seen in "Thus the World Was Created" chart that the Lunar Mercury Planet Symbol (which is 8) is associated with "Lunary things" like the element Water, the White alchemical stage, and thus is "female."

The Solar Mercury Planets Symbol (which is 9) is associated with Solary thing like the element Fire, the Red alchemical stage and is thus "male."

If **8 is the female parent**, and **9 is the and male parent**, it seems logical that **10 is the King**, the "ultimate power" that rules over the realm of single digits. (As the whole Monas Hieroglyphica is about numbers, it makes sense that this maxim of the flowing ribbons is about numbers as well.)

37

The tip of a sharp spear, arrow, needle, thorn, or bee's stinger
is **one** point
(ouch)

The "sharp, stable point"
is a pretty obvious metaphor for
"**one**,"
as in "one point."

In his Letter to Maximillian, Dee says the Monas symbol is like the symbol of "**Mercury, (fortified by a sharp point).**" The traditional symbol of Mercury is the Monas symbol without the Aries symbol attached. This "sharp point" seems to be the point at the base of its spine, where it connects the Aries symbol.This "**1 poin**t" also happens to be "**point 1**" in the process of counting the 10 points of the spine.

one
sharp
point
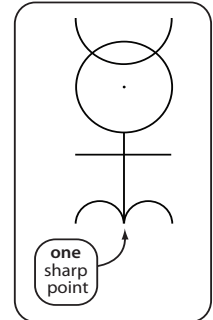
The word Mercury has been used so many different places here; it seems to be playing many roles (sometimes confusingly). However, it's actually quite appropriate, as Mercury is a "mercurial" or "**changeable**" thing in alchemy.

"Mercurius" refers to not only the "prima material" sought at the beginning of the work, but also to the "ultima material," the finished goal of the work (the Philosophers Stone.) It also presents "everywhere and at all times" during the work. The alchemists called it a Hermaphrodite as it incorporates both the male and female principles.   (Abraham, p.124-128)

Mercury is a pretty diverse and can take on any number of forms.
It's an alchemical chameleon.
With these numerical puzzle-parts, the maxim seems to read:
"that changeable thing, Mercury, plus one, equals **"8, 9, or 10."**
But that doesn't make much sense.
So instead, let's read the maxim as if it were a musical "round" of three separate sentences:

If Mercurius is "all the planets," for the first "round" we might first see it as the number 7.
Mercurius (7), plus a sharp, stable point (1), becomes the "female parent" (8).

Now, that "changeable" Mercury has become the number 8.
The next "round" would be:
Mercury is (8), plus a sharp, stable point (1), becomes the male parent (9).

Now, that "changeable" Mercury has become the number 9.
The final "round" would be:
Mercury (9), plus a sharp, stable point (1), becomes the King (10).

In short, the maxim
of the flowing ribbons says:

$$7 + 1 = 8$$
$$8 + 1 = 9$$
$$9 + 1 = 10$$

Let me historically substantiate my analysis that is maxim is actually a "round" of three separate sentences (or equations).

A round is a special kind of a canon in which two or more voices (or instruments) sing (or play) the same music starting at different times. When each voice finishes, it starts at the beginning again, so piece can go 'round and 'round. Often it's called a circular canon. Everyone is familiar with the round "Row, Row, Row your boat."

> Row, Row, Row your boat,
> Gently down the stream,
> Merrily, Merrily, Merrily, Merrily,
> Life is but a dream.
> Row, Row, Row your boat,
> Gently down the stream,
> Merrily, Merrily, Merrily, Merrily,
> Life is but a dream.
> Row, Row, Row your boat,
> Gently down the stream,
> Merrily, Merrily, Merrily, Merrily,
> Life is but a dream.
> Row, Row, Row your boat,
> Gently down the stream,
> Merrily, Merrily, Merrily, Merrily,
> Life is but a dream.
> ...

> Miss Lucy had a steamboat
> The steamboat had a bell,
> Miss Lucy went to heaven and the
> Steamboat went to…
>
> Hello operator
> Please give me number nine
> And if you disconnect me
> I will chop off your…
>
> Behind the 'fridgerator
> There was a piece of glass
> Miss Lucy sat upon it
> And she got her big fat…
>
> Ask me no more questions
> I'll tell you no more lies…

Dee's numerical round is actually more like "Miss Lucy had a Steamboat," in which each stanza is connected to the previous stanza with a clever wordplay.

You don't hear many "rounds" on the radio these days, but they were popular throughout Europe from around 1300 to 1900. (That's pretty long when you consider how long Disco lasted.)

Medieval music scholar Frank L. Harrison, of Oxford University found a type of a round called a "rondella" that dates back to 1065.

One of the oldest surviving English rounds (from around 1400) is "Sumer is Icumen In," (or "Summer is A'coming In"), which is for 4 voices. It's known that even Queen Elizabeth 1 knew how to play that ditty.

Musical researchers John E. Stephens and Jill Vlasto recently found a Court songbook from the early years of King Henry VIII's reign (1509–1547) that contained 20 rounds and canons. Vlasto also discovered a manuscript dated 1580 by Thomas Ravenscroft which recorded 48 different rounds.

In the 1700's, Hayden, Beethoven, Mozart, Schubert, and Brahms all wrote rounds and canons. Johan Pachabel wrote a canon for string orchestra that is still played at weddings today.

Johan Sebastian Bach wrote many canons including a work called "Musical Offering," in which one line of the melody is reversed in time and pitch from the other. This type of "musical palindrome" is often called a "crab canon," as a crab walks sideways, back and forth.

The round may be a clever device Dee used to hide his clues, but the song "7+1 =8, 8+1 = 9, 9+1=10" seems like something you would learn in kindergarten, not puzzle the King of the Holy Roman Empire with. What the heck does it mean?

Well, actually, we've already seen these 3 equations in Dee's work.

Recall that in the Artificial Quaternary, after Dee obtained 8 by "summing $1, 2, 3, 2$," he inexplicably broke 8 into $1 + 7$. There's the first round: **7+ 1 = 8**.

(A further clue is that he inexplicably late breaks down 7 into 4 and 3. This is the same thing he presents Theorem 6, where he sees the Cross as Ternary and also as Quaternary, thus manifesting the "Most Excellent Septenary." This 4 and 3 is also an expression of "Quaternary Rests in the Ternary.")
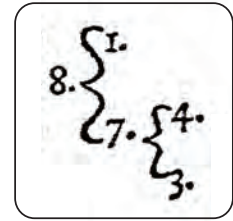
Dee sings the **8 + 1 = 9** stanza graphically in the Thus the World was Created are by placing the " HORIZON OF ETERNITY" above the Octonary of numbers (curiously grouped as $1, 2, 3, 4$, and $5, 6, 7, 8$).

And finally, Dee sings the **9 + 1 = 10** stanza in his Monas symbol, which has 9 parts and 10 points.

All this seems to confirm that we're on the right track, but it **still** doesn't explain what he's trying to say by all of this.
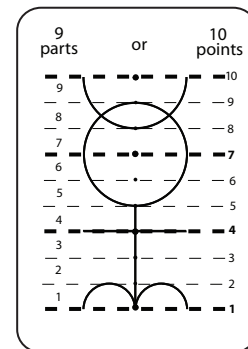
Kelley squeals

Clues to what Dee had in mind can be found in the writings of:
Henry Cornelius **Agrippa** of Nettesheim (ca. 1486–1535),
Phillipus Aureolus Theophrastus Bombast Von Hohenheim
(who called himself "**Paracelcus**" for short) (1493–1541),
and strangely enough, Dee's scryer Edward **Kelley** (1555–1597).

Agrippa cites the Neoplatonic philosopher Proclus (ca. 410–485) as expressing:
**"All things,
with the number 10,
and by the number 10,
can make a round."**

He calls 10 "as circular as unity, because being heaped together, returns into a unity from whence it had its beginning… like water returns to the Sea from whence it had its beginning…"

(Agrippa, p. 287-8)

Paracelsus in *Aurora of the Philosophers* is a little more detailed about the whole process, but it's somewhat hidden in his philosophical/religious lingo:

> "Magic, it is true, had its origin in that Divine Ternary and arose from the Trinity of God.
>
> For God marked all His creatures with this Ternary and engraved in its hieroglyph on them with His own finger.
>
> For the Ternary, with the magical Quaternary, produces a perfect Septenary, endowed with many arcana and demonstrated by things which are known.
>
> When they Quaternary rests in the Ternary, then arises that Light of the World on the horizon of eternity, and by the assistance of God gives us the whole bond.
>
> Then when the Quaternary and the Ternary mount to the Denary, is accomplished their retrogression or reduction to unity.
>
> Herein is comprised all the occult wisdom of things which God has made plainly manifest in man, both by His word and by the creatures of His hands, so that we may have a true knowledge of them."

Dee was a big fan of Paracelsus and owned most of his writings, including the *Aurora of the Philosophers*. (Robertson and Watson, p. 200, book 177)

Some people feel Dee's scryer Edward Kelley was a gifted clairvoyant who really spoke to angels. Others think he was a charlatan.

It is quite clear in what follows that he was not above "borrowing" someone else's ideas to make them seem like his own. (The English word plagiarism wasn't coined until the 1600's, from the Greek root word *plagium*, a kidnapping). In Elizabethan times, uncredited borrowing was perhaps more acceptable than it is today, but by comparing the sentences of these texts, it's obvious from reading Edward Kelley's *Theatre of Terrestrial Astronomy*, that he had Paracelsus' *Aurora of the Philosophers* on the desk in front of him as he wrote.

> "Magia derived its origin from the doctrine of the Divine Ternary and the Trinity of God.
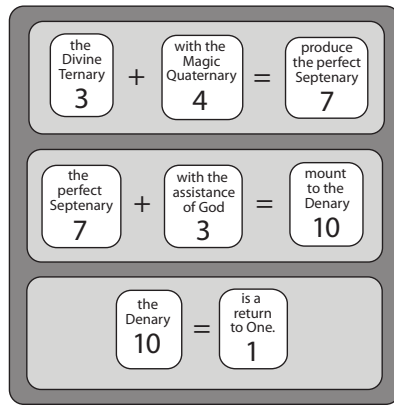>
> For God has stamped and sealed all created things with his this character of Trinity, as a kind of hieroglyphical writing, whereby His own nature might be known.
>
> For the number 3 and the magic number 4 make up the perfect number 7, the seat of many mysteries.
>
> And seeing that the Quaternary rests in the Ternary, it is a number which stands on the horizon of eternity, and doth exhibit everything bound in God in us, thus including God, men, and all created things, with all their mysterious powers.
>
> Adding three, you get 10, which marks the return to unity.
>
> In this arcanum is concluded all knowledge of hidden things which God, by His word, has made known to the men of His good pleasure, so that they might have a true conception of Him."

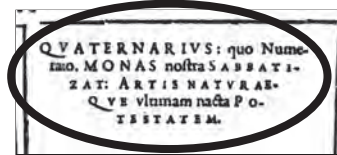| the Divine Ternary **3**	+	with the Magic Quaternary **4**	=	produce the perfect Septenary **7**
the perfect Septenary **7**	+	with the assistance of God **3**	=	mount to the Denary **10**
the Denary **10**	=	is a return to One. **1**		

My intention here is not to highlight Kelley's copying, but to show how he was influenced by Paracelsus. This chart simplifies what they are both saying:

When the concept of "10 is a return to 1 is incorporated, it's easy to see this as a musical round that cycles 'round and 'round.

$$1 + 3 = 4$$
$$4 + 3 = 7$$
$$7 + 3 = 10$$
$$10 \text{ is a return to } 1$$
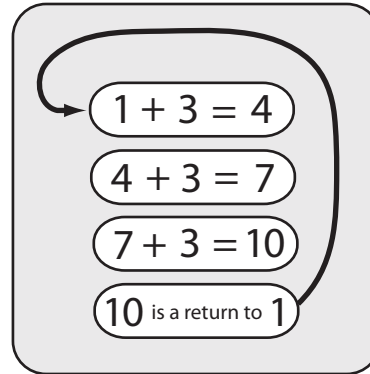
This appears to be the song Dee is singing in the upper-right quadrant of the "Thus the World Was Created" chart, by cryptically referencing:

1 (Monas),
4 (Quaternarius),
7 (Sabbatizat) and
10 ("ultimate POWER of NATURE'S ART)

QVATERNARIVS: quo Numero, MONAS noftra SABBATI-ZAT: ARTIS NATVRAE-QVE vltimam nacta Po-TESTATEM.

(The idea that 10 is a return to "1" really reinforces the idea that it is the "ultimate power.")

This "musical round "is more like the "Row, Row, Row Your Boat" kind of round than the "Miss Lucy Had a Steamboat" kind of round, as it returns back to the beginning and starts over again.

"**1**+3=4, +3=7, +3=10, which is a return to **1**, +3=4, +4=7, +3=10, which is a return to **1**,+3=4, +4=7, +3=10, which is a return to **1**,+3=4, +4=7, +3=10, which is a return to **1**,+3=4, +4=7, +3=10, which is a return to **1**, +3=4, +4=7, +3=10, which is a return to **1**..."

Actually, it's a lot like this "round sentence"about girl detective Nancy Drew doing endless self-portraits:

"Nancy **Drew** a picture of Nancy **Drew** a picture of Nancy **Drew** a picture of Nancy **Drew** a picture of Nancy **Drew** a picture of Nancy **Drew** a picture of Nancy **Drew** a picture of Nancy **Drew...**"
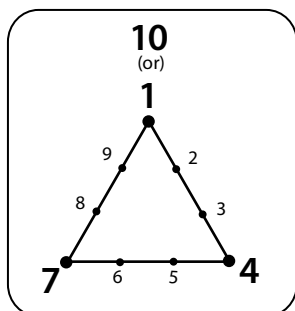
Or even the way notes repeat themselves in musical scales:

"do re mi fa sol la ti **do** re mi fa sol la ti **do** re mi fa sol la ti **do** re mi fa sol la ti **do** re mi fa sol la ti **do** re mi fa sol la ti **do** re mi fa sol la ti **do** re mi fa sol la ti **do** re mi fa sol la ti **do...**"

As we've seen, the centerpoints of the 4 parts of the Monas symbol express this same idea. But with point 1 at the bottom and point 10 at the top, the Monas symbol doesn't really express this "10 is a return to 1" idea so critical in perceiving it as a "round."

We could incorporate this idea if we somehow curved the Monas symbol around in a circle so that point 1 and point 10 became the same point.
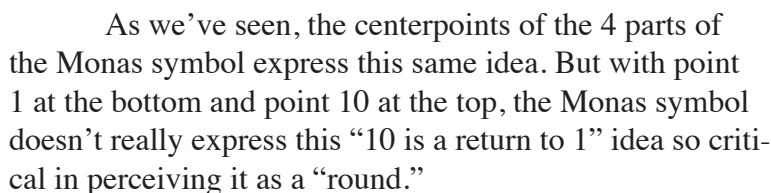
Backbending Dee's straight, proud Monas symbol like this might seem like unjust maltreatment, but to understand what Dee is trying to say, we actually have to bend it a little bit more!

Monas symbol
curved into a circle

Monas symbol
bent into a triangle

When bent into a triangle, the centerpoints of the Aries, Cross, Sun and Moon symbols are **all located at corners.**
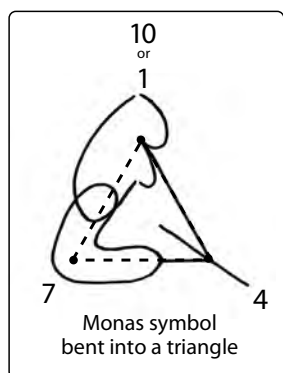
Suddenly the Monas symbol has become Dee's signature!

Now you can see why Dee calls it the "Symmetry of the Decad." It's not "two part" reflective symmetry like a mirror or a see-saw, but "three part" symmetry like the shape of a stable, equilateral triangle

Dee worked this "1, 4, 7, 10 Symmetry of the Decad" pattern into the fabric of the Monas in another clever way: in the organization of his Theorems!

You might have noticed that the beginning Theorems are all short (a few sentences each) and the later Theorems get quite lengthy (a few pages each). Dee crafted it this way for a reason. Look at the first 4 pages of the Monas.

The first page starts with Theorem 1,
The second page starts with Theorem 4.
The third page starts with Theorem 7.
And the fourth page starts with Theorem 10.

Dee' depicts the "1, 4, 7, 10" Symmetry of the Decad in the page layout of his book.
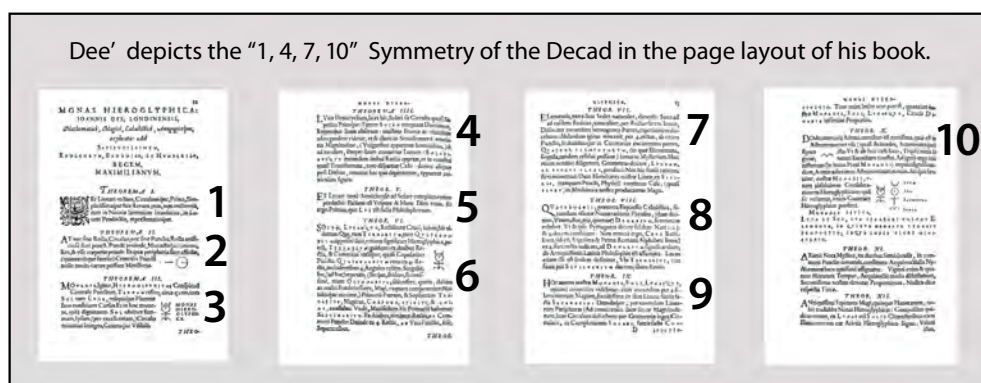
There's the "1, 4, 7, 10 Symmetry of the Decad" pattern!
The layout of the book graphically depicts the very dictum it professes!

Actually, Theorem 10 isn't *exactly* at the top of the page. There is some "carry-over" from Theorem 9 above it. This is simply Dee's way of not being too obvious. But he left some confiming clues to let us know we're on the right track.

Notice what the the "carry-over "word is : CONJUNCTIO, split as CON and JUNCTIO ("con" means "together" and "jungere" means "to join").

Not only is this a word that depicts itself (it is a conjunction of two words), but Dee uses it to conjoin the two parts of Theorem 9 that are on separate pages.

That last, orphaned sentence of Theorem 9 reads:

*"Thus it is not possible to hide how much
the Denarian symmetry of the Cross of our Monad
usefully serves the Sun and the Moon."*

The "Denarian symmetry," on one level, is the Roman numeral for 10, namely the symmetrical X-shape, which Dee saw as as a form of a cross. But on another level, it means the "1, 4, 7, 10" Symmetry of the Decad.
We might graphically express it this way:

Theorem 10
or
Theorem 1

Theorem 9   Theorem 2

Theorem 8   Theorem 3

Theorem 7   Theorem 6   Theorem 5   Theorem 4

Theorems at the tops
of the first 4 pages
of the *Monas*
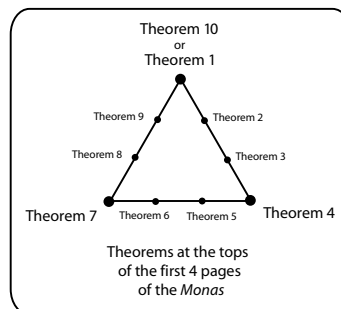
(This "1, 4, 7, 10" page-layout clue seems to be one of the "other things" that Dee asks Gulielmo Silvio to "carefully copy.")

Just as 9 is the "Horizon" number," the final sentence of Theorem 9 is a "Horizon" sentence. It is on the "edge" of Theorem 10, which conains an important maxim (it's written in all capital letters),which begins:

"THE ELEMENTS OF THE SUN AND MOON OF THE MONAD,

IN WHICH THE DENARIAN SYMMETRY IS STRONG..."

(Note that this is essentially a repeat of the final sentence of Theorem 9.)

"... WANT TO BE SEPARATED,

AND THIS IS DONE WITH THE AID OF FIRE."

On one level, this refers to the Aries symbol which, when added to the Sun, Moon, and Cross, makes the full Monas symbol. On astrological level, Aries is one of the "fiery triplicity"comprised of Aries, Leo, and Sagittarius. (Dee calls it "TRIPLICITATIS PRIMAE," "prime or first triplicity" in a "carry-over "word at the end of Theorem 21.)

But "FIRE" has an even deeper meaning which relates specifically to the "point"where the Aries symbol contacts the bottom of the Cross. Can you guess what Dee is metphotically calling a "point" here? Something that somehow sparks the the "1, 4, 7, 10" pattern ((Denarian symmetry of the Cross) **and** the idea of "opposites" (Sun and the Moon)?

To summarize, piecing together these various clues gives a clear picture of Dee's Symmetry of the Decad": **"1, 4, 7, 10, which is a return to 1"**
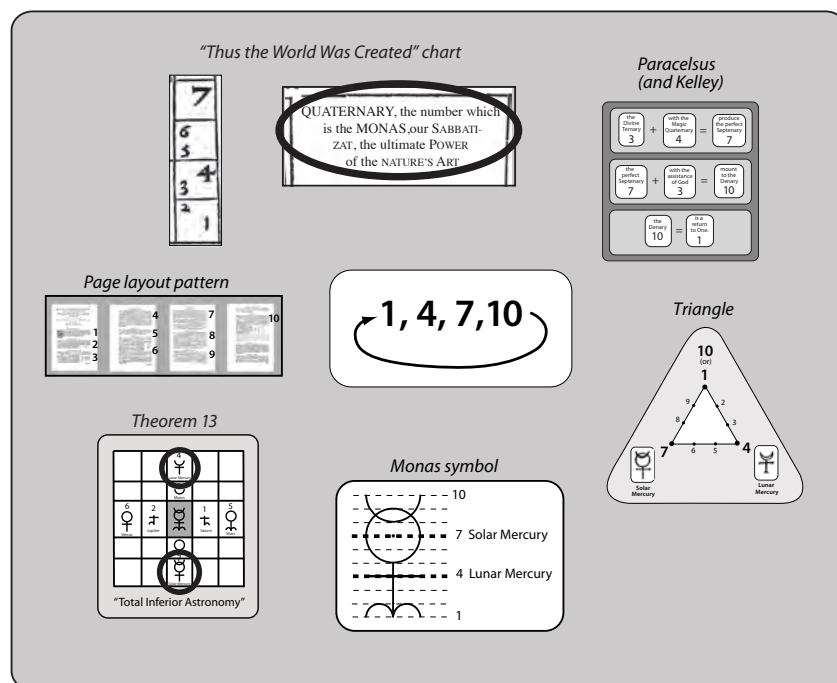
Again, this is all very nice and symmetrical and philosophical, but it still doesn't bring us any closer to what Dee **means** by all these mathematical machinations. Perhaps there's a clue in the Artificial Quaternary chart.

45

Artificial Quaternary Chart

This chart starts with a bold proclamation by Dee:

"Our Numbers have such Dignity that to violate their Laws
would be a Sin against the Wisdom of Nature.
Indeed, these Laws announce with authority
the certain and Fixed Limits that Nature wants to teach us
(in the examination of its greatest Mysteries)"

That's pretty strong talk. Dignity, Laws of Number, Sin, Wisdom of Nature, Fixed Limits, Mysteries. What's this all about? The only obvious thing is that it somehow involves numbers.

Note that Dee's pronouncement has brackets on each side, as if it is connecting the Artificial Quaternary (on the preceding page) to the information in this chart.

Dee tells us in the Text of Theorem 23 that this chart shows "how our Numbers Originate" in 3 ways:

Virtue (Virtus), "in the ordering of the STEPS of the Power and Virtues of Things"
Weight (Pondera), "in the WEIGHING OF ELEMENTS"
Time (Tempora), "in marking the MEASURES OF TIME"

The "Artificial Quaternary" chart
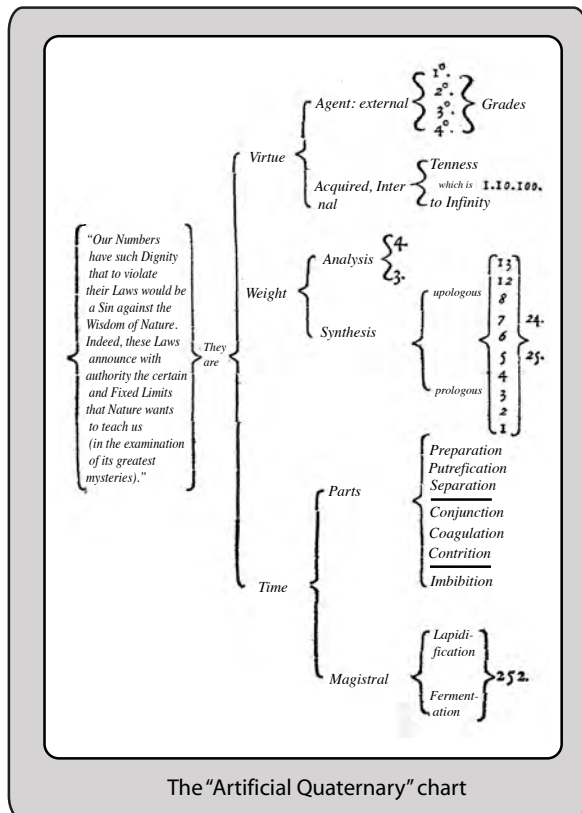
In the "***Virtue***" category are two subcategories. "*Agent: external*" is associated with "4 Steps." The digits 1, 2, 3, and 4 have small circles next to them. To the modern eye, this reads as "degrees," either in temperature or angles. But Lord Kelvin didn't invent the thermometer until around 1890, and small circles weren't used to denote angular degrees until after Dee's time.

The second subcategory is labeled "*Acquired, Internal*" and summarizes how Dee saw "Tenness" in the sequence "1, 10, 100… to Infinity."
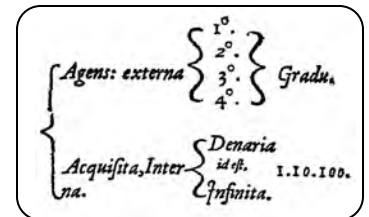
What does "4 things" have to do with "external" and "10 things" have to do with "internal"?

Here's an example where an important clue is lost in the translation from Latin to English. I give you a hint. There's an example of Dee's "Tzmiruph" or a "jumbled-letter clue" in the original Latin letters of:

Agens: externa
Acquisita, Interna

(I give you another clue. The solution is a Latin phrase.
This makes it harder, but it's a phrase that Dee used quite frequently)

QUATERNARIUS INTERNARIO CONQUIESCENS
or the
"Quaternary Rests in the Ternary"

(This revelation might not bring much closer to what this phrase actually means, but it does tip us off to the idea that there may be additional jumbled-word clues in the Monas.)

Next, the "*Weight*" category has two parts, "*Analysis*," which seems to echo the same "Quaternary Rests in the Ternary" idea, and "*Synthesis*." Analysis means studying the various parts that make up a whole. Synthesis is putting parts together to study a whole.

The Greek word "*upologous*" means the "second, larger term of a fraction and "*prologous*" means the "first, smaller term" of a fraction. The "fraction line" that we use today wasn't popularized until around the 1200's. The Greeks would put the numerator first, followed by an accent mark, then the denominator next, followed by 2 accent marks. However, the Greeks didn't look fractions the way we do today. For example, 3'4" to them meant what 4/3 means to us today. So in modern-day terms, upologous would be our denominator, and prologous means our numerator.

The fact that he wrote these words in Greek and reversed them (by printing the denominator above the numerator) suggests Dee wants us to study part-to-part ratios instead of part-to-whole fractions. In short, 3/4 (or 75%) does not equal 4/3 (or 133.33...%), but the ratio of "3 things:4 things" is the same as the ratio of "4 things:3 things."

Just to the right of this is a listing of the digits from *1–8*, numbers *12* and *13*, and numbers *24* and *25*. These are numbers Dee shows in the Thus of the World was Created chart. Notice that 9, 10, and 11 are conspicuously absent. As we just saw that **10** featured prominently just above in the Virtue category, we might say that **only 9 and 11 are "missing."** (There's a good mathematical reason for this, as we shall see.)
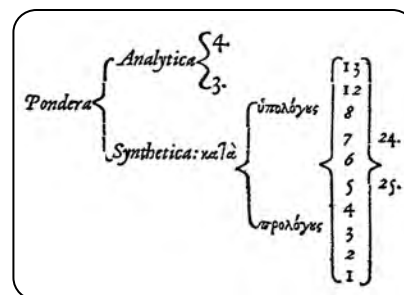
The third category is "*Time*." The subcategory "Parts" is a listing of various alchemical procedures. (In Dee's original Latin text, they are merely the abbreviations.) There are 7 of them, although the way they are listed, it seems like there are 9 with the 4th and 8th procedures "missing," with only a horizontal line in their place.

The second subcategory of "Time" is labeled Magistralia and incorporates Lapidification and Fermentation. (In Dee's original Latin, it's "*Magistralia*," incorporating "*Lap.*" and "*Ferm.*")

Lapidificaton means transforming something into a stone. The Oxford English dictionary says that the original meaning of "ferment" is "leaven or yeast" adding that "in Alchemy it is sometimes applied to the philosophers stone."   (OED, p. 985)
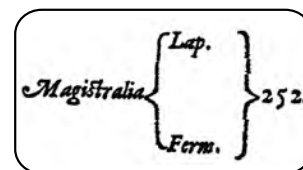
In his 1947 translation of the *Monas Hieroglyphica*, J. W. Hamilton-Jones sees this as "fermentation of the Philosophers Stone." In the 1690 translation by the "Anonymous," it is seen as "fermentation of ye Stone."

The Latin word *Magistralia* comes from the Latin word *magister* meaning "master." From it, we derive two modern words: "magisterial," meaning "having great authority" and "magistral," meaning "having to do with a master."

In Theorem 16, Dee multiplies to two V-shapes together (Roman numeral 5's) to derive 25. He multiplies two L-shapes (Roman numeral 50's) together to get 2500. These two results are close friends with 250, but not really with 252.

Continuous doublings of 2 brings you to 256 (2, 4, 8, 16, 32, 64, 128, 256), but not 252.

It's pretty clear that 252 is pretty darn important in Dee's mathematical cosmology. In Theorem 17, he derived 252 from "20 + 200 + 10 +21 + 1," adding "there are two other logical ways that we can draw forth this number from our premises."

252   Here, he appears to be calling it a "Master" number and the "Philosophers Stone." What's the big deal about **252**?   **???**

Most of the Monas seems to deal with single-digits. Why is this 3-digit number all that important? To help figure this out, I did what any modern day researcher would do:

I googled 252.

I came across this quote, which sounds as obscure as Dee writing:

"Thus by experimental evidence we may identify the electron with the volume of the regular, unit-vector-radius-edge tetrahedron, the simplest symmetrical structural system in Universe. We may further identify the electron tetrahedra with the maximum possible symmetrical aggregate of concentrically packed, unit-radius spheres symmetrically surrounding a single nucleus – there being 12 new potential nuclei appearing in the three-frequency shell of 92 spheres, which three-frequencies shell, when surrounding embraced by the four-frequency shell of 162 spheres, buries the 12 candidate new nuclei only one shell deep, whereas qualifying as full-fledged nuclei in their own right requires two shells all around each, which 12, newborn nuclei event calls for the fifth-frequency shell of 252 spheres."

This quote comes from Buckminster Fuller, a man I consider the John Dee of the 20th Century. Like Dee he was interested in geometry, number, navigation, astronomy, architecture,as well as grander concepts like "Humanity" and "Universe."

We must leave Dee's time for a while.
Sherman, reset the Way-Back machine
to a kindergarden in Milton, Massachusettsin the year 1899.

Bucky's Kindergarten discovery

Buckminster Fuller recalls his unusual childhood:

"I was born cross-eyed on 12 July 1895. Not until I was four-and-a-half years old was it discovered that I was also abnormally farsighted... Until four-and-a -half I could only see large patterns—houses, trees, outlines of people–with blurred coloring.

While I saw two dark areas on human faces, I did not see a human eye or a teardrop or a human hair until I was four...

I was sent to kindergarten before I received my first eyeglasses. The teacher, Miss Parker, had a large supply of wooden toothpicks and semidried peas into which you could easily stick the sharp ends of the toothpicks. The peas served as joints between the toothpicks.

She told our kindergarten class to make structures. Because all of the other children had good eyesight, their vision and imagination had been interconditioned to make the children think immediately of copying the rectilinearly frame structures of the houses they saw built or building along the road.

…In my poor-sighted, feeling-my-way-along manner I found that the triangle—I did not know its name—was the only polygon—I did not know that word either—that would hold its shape strongly and rigidly.

So I naturally made structural systems having interiors and exteriors that consisted entirely of triangles.
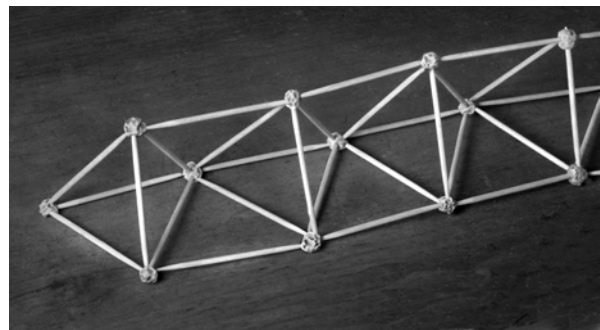
A re-creation of part of Bucky's structure made from toothpicks and semi-dried peas.

Feeling my way along I made a continuous assembly of octahedra and tetrahedra, a structured complex to which I was much later to give the name contracted name "octet truss." The teacher was startled and called the other teachers to look at my strange contriving."

(Fuller, *Synergetics* 2, pp.231-233)

When Bucky recounted this event during a lecture in 1975 (at age 80), he added,

"All the other kids, the minute they were told to make structures, immediately tried to imitate houses. I couldn't see, so I felt. And a triangle felt great! I kept going 'til it felt right, groping my way..." (Fuller, in Edmondson, p. 141)

Strange as it seems, what Bucky intuited in his kindergarden class and later embellished upon lies at the heart of what *Monas Hieroglyphica* is all about. Bucky and Dee came across the same things in their studies of Nature. (Quite independently, as Bucky was not aware of Dee's works). Before explaining this this seemingly implausable similarity, let's first paint a quick picture of Bucky.

A Brief Biography of Bucky
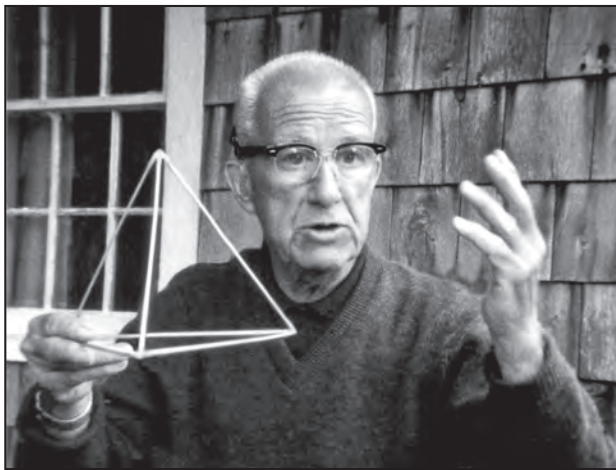
(Photo courtesy of the Estate of R. Buckminster Fuller)

Richard Buckminster Fuller was an inventor, philosopher, engineer, and architect. Called by some the Leonardo Da Vinci or Benjamin Franklin of the Space Age, he was one of the most important thinkers of the 20th century.

He worked globally and worked fervently suggesting ways "to make man a success in Universe." Let's take a very brief tour of his family heritage and highlights of his life from 1895-1983.

Bucky's heritage

In 1630, ten years after the Pilgrims landed in Plymouth. **Thomas Fuller**, a lieutenant in the English Navy, was visiting New England on a furlough. Seeing unbounded opportunity, he decided to become part of the blossoming settlement.

In 1787, over a century later, Fuller's grandson **Reverend Timothy Fuller**, a graduate of Harvard, represented Massachusetts in the Federal Constitutional Convention in Philadelphia. Because the proposed Constitution didn't ban slavery (one fifth of the population of America were slaves) the freethinking Reverend Fuller voted against it. (Seiden, p. 2)

In the early 1800's, his son **Timothy Fuller Jr**, and grandson **Reverend Arthur Buckminster Fuller** both attended Harvard. The Reverend Fuller was a "staunch abolitionist" and died fighting in the Civil War.

Another notable member of the Fuller clan was **Margaret Fuller**. With her close friend Ralph Waldo Emerson, she founded "The Dial" a Transcendentalist literary magazine.

Bucky's father, **Richard Fuller** graduated from Harvard in 1883 and traveled the world as an importer of leather and teas.

Bucky: the early years

Bucky was born on July 12, 1895 in Milton Massachusetts. During the summers of his youth the Fullers vacationed their privately owned island in Maine. Bucky was an avid sailor Local fisherman learned and learned the art of navigation from the "Down East" fishermen.

Fuller attended Milton Academy, then Harvard in 1913. But because of too much partying and missing midterms he was expelled. Bucky's father had died, but his mothers and uncles, outraged by Bucky's "scandalous behavior," sent him off to work at a new textile mill in Canada owned by a distant relative.

Most people would have considered it glooming exile, but Bucky viewed it as a challenge to learn about engineering and industry.

The following year Harvard let him back in, but he was promptly expelled for a second time, due to more partying and "lack of ambition." In truth Bucky was more into exploring novel ideas than "memorizing facts." (Sieden, p22)

This time Bucky moved to Manhattan and worked in the corporate office of the meat distributing firm Armour and Co. In this busy metropolis, he started thinking about ways in which people could live more efficiently in closely packed communities.

In 1917, Fuller married Anne Hewlett (his wife of 66 years). He also enlisted in the US Navy Reserve. As lieutenant and communications officer aboard the USS George Washington, he was involved in the first transatlantic radio transmission of voice messages. (Sieden, p. 50-52)

After the war, he and his father-in-law built houses made from ad unbreakable, yet lightweight, fireproof compressed fiber block they invented.

Buckminster Fuller, ca. 1917

1927
a turning point for Bucky

The year 1927 was a rollercoaster ride for 32-year-old Bucky. He was president of Stockade Building Systems and his daughter Allegra had just been born. Suddenly, change was in the wind. Financial difficulties forced Bucky's father-in-law to sell his stock in the company. Before the month was out, the company was sold and Bucky was out of a job. Soon all of his savings were gone and he was falling further and further into debt. Down and out, he started drinking and carousing in the streets of Chicago.

One cold fall night he walked to the shore of Lake Michigan and considered jumping in, swimming as far out as he could far out in the cold water and ending his life. Suddenly something clicked and he realized "You don't have the right to eliminate yourself. You do not belong to you. You belong to the Universe." (Sieden p.88) He decided to start a fresh new life, think less about himself, and assess how he could best help all humanity.

He rented a single room in a cheap hotel in the city's ghetto area. It had with one closet and an alcove with a stove and a sink. He took a vow of silence for 2 years, speaking only to his wife and daughter.

He studied great thinkers like DaVinci and Gandhi. He studied astronomy, physics, biology, and mathematics, to see how nature expressed itself. He sensed that nature had certain "pattern integrities," that might not be detectable by the physical senses, but that might be expressed with tangible models.

He set out to find "**Nature's one comprehensive coordinate system**" the geometry of the universe. (And found it; more on this later).

Bucky designs cars, homes, maps, and domes

He wrote his first book, 4-D Timelock. He invented a mass produced dwelling unit that consisted of a hexagonal living area suspended above ground by cables attached to a central mast. It became known as the Dymaxion House, (a combination word made from "dynamic," "maximum," and the scientific word "ion")

Five years later, Bucky designed the Dymaxion Vehicle, a 3-wheeled automobile. The two front wheels had the power, but the solo rear wheel did the steering. Because of the triangulation of its 3 tires, it could turn on a dime. Mimicking the natural shape of birds and fish, it was much more streamlined and aerodynamic than the boxy cars of the 30's.

Bucky felt that looking most maps of the world was like looking at one's reflection in an ex warped amusement-park mirror. So he designed his Dyxmaxion Map of the world, geomerically based on triangles, in which the continents are displayed with in their actual proportions to one another..

During World War II, Bucky became the Director of Mechanical Engineering for the US Board of Economic Welfare. After the war, Bucky was teaching at MIT and invented the geodesic dome.

A geodesic is the curve that is the shortest line between two points on a sphere or curved surface other. Geodesy is the study of the size and shape of our spherical planet. In Greek, *geo* means "Earth" and *daiesthai* means "to divide."

For centuries St. Peter's Cathedral in the Vatican held the record for having the world's largest clear-span dome, at 150 feet in diameter. In 1957 Bucky more than doubled that record with his 384-foot diameter dome built in Baton Rouge, Louisiana.

For the 1967 World's Fair in Montreal, Bucky designed a 250 foot diameter "biosphere" of steel rods and transparent acrylic panels.

In the 1960's and 1970's, Bucky gave hundreds of lectures every year at conferences and colleges all over the world. By 1971 he circumnavigated the globe 37 times.

(Bucky's chronology, Seiden, p. 419-437)

Bucky's masterwork: Synergetics 1 and 2

In 1975, Bucky wrote his 876-page opus **SYNERGETICS**, *Explorations in the Geometry of Thinking* synthesizing his discoveries about Nature's coordinate system.

Realizing he still had more to say, he followed it in 1979 with his 592-page *Synergetics 2*.

Call Me Trimtab.

In 1983, his wife Anne fell into a coma in a Los Angeles hospital. Bucky was given a room down the hall so he could be near her. Suddenly he suffered a massive heart attack and died. Anna died the following day.

Bucky and Anna lie side by side on a tree-shaded hillock in the beautifully landscaped Mount Auburn Cemetery (Cambridge, Massachusetts) surrounded by several dozen other Fullers.

His tombstone reads, "Call me Trimtab."
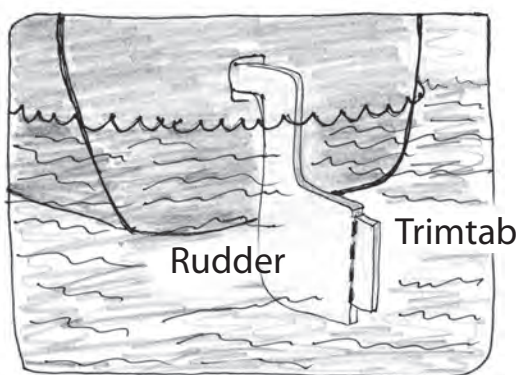
It takes a lot of force to turn the rudder of a large ship traveling with great momentum. A trimtab is a mini-rudder hinged to the tail edge of a ship's main rudder.

When the trim tab is turned slightly, it creates a low-pressure area in the water, allowing the main rudder to be turned with much less effort.

Bucky felt he was "one little man" whose thinking could influence the thinking of others, who, in turn, would help steer humanity in a positive direction.

Both Bucky and Dee sought to understand how Nature operates

        I see John Dee as an Elizabethan Trimtab. He was one small individual who tried to help steer England's ship of state. He lobbied to have England's calendar changed so it was realigned with the sun's movement. He petitioned the Queen to settle the New World to ease the tense religious animosity in Elizabethan England. And he wrote the *Monas Hieroglyhica* and designed the John Dee Tower to share his understanding of numbers and geometry– clues to how Nature operates.

What Bucky discovered

        To understand what Dee found, we can learn from what Bucky found. They both wewe looking for the same thing so it's not unusual that they might find the same thing. Much might have changed between the 1500's and the 1900's, but the "Laws of Nature" have stayed the same.

        Studying the almost 1500 pages of "idiosyncratic, hyphenated prose" of Bucky-speak in the *Synergetics* volumes is a daunting task. Fortunately, one of his students, Amy Edmondson has summarized his ideas in ***A Fuller Explanation,*** *The Synergetic Geometry of R. Buckminster Fuller.*

        I have attempted to boil everything down even further, to the bare essentials, and present it in a concise visual style, so realize that much in-depth info has been left on the cutting room floor. I encourage readers to study Bucky's primary text for elucidations and the final word. (The *Synergetics* texts and *A Fuller Explanation* are both on the web)

<div align="center">

As a framework, I will use the "10 Considerations"

which Bucky presents in Synergetics 2

(p. 236-242+).

They synopsize his thought process in his discovery of

"Nature's coordinate system."

</div>

Bucky's
"10
Considerations"

Consideration 1. Energetic Vectors

Bucky starts off, "**I first determined to employ only vectors as lines**." A vector defined is a geometric object with a length and a direction. It's frequently shown as a line segment with an arrow that goes from initial point A to terminal point B. Vectors don't extend to infinity like lines extend to infinity, or as Bucky puts it, "Vectors always have a unique length" and "are inherently terminal." In short, it's a finite line.
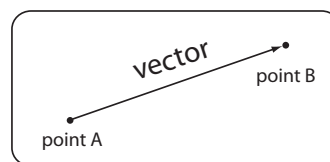
Consideration 2. Avogadro's Constant Energy Accounting

The Italian scientist Amedeo Avogadro (1776-1856) discerned around 1820 that, under constant conditions of heat and pressure, **a given volume of gas will always contain the same number of molecules**. This suggested to Bucky that the geometrical arrangement of these molecules could be demonstrated physically.

Consideration 3. Angular Constancy

If vectors represent the energy relationship between molecules, and "all the conditions of energy are constant throughout the volume, vectors must interact at the same angles. If the conditions of energy are consistent throughout the volume of molecules, the energy vectors "**must all interact at the same angles."**

Consideration 4. The Isotropic Vector Model

Bucky asked himself, "Can I make a vector model of this Avogadro generalization?"

Then he remembered he had already done so in "that kindergarten event in 1899, when I was almost inoperative visually and was exploring tactilely for a structure that would hold its shape." (Bucky Synergetics 2, p. 238)

When lecturing, Bucky would frequently give his "necklace" demonstration. Around his neck he would drape a large dangling necklace made from 10 or more 10-inch dowels connected with short, flexible segments of rubber tubing. He would remove the dowels one at a time, and the necklace would shrink in circumference.

Soon the necklace became a septagon, then a hexagon, and then a pentagon. Each time Bucky would try to attempt to show them as them as symmetrical shapes, but they were never perfectly symmetrical. When he got down to 4 dowels, he would show it as a wobbly square. He even showed how it could be folded into a bundle of 4 parallel sticks. He removed one more dowel and suddenly he had a shape that could not be crushed: **the triangle.**
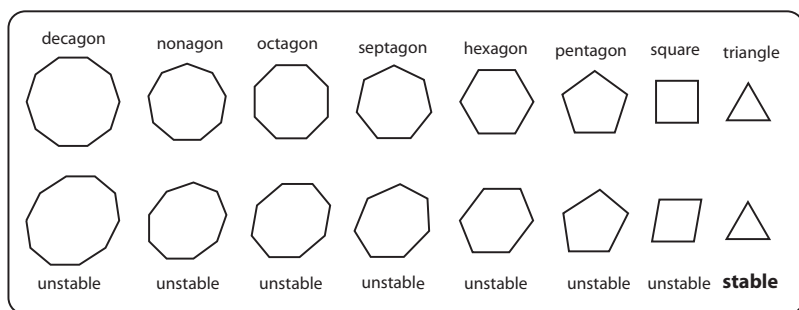
He would gush forth emphatically,

"**Any polygon with more than three sides is unstable.**

Only the triangle is inherently stable…
If we want to have structure, we have to have triangles."
(Synergetics 1, 609.01 p. 319, 610.12 321)

decagon · nonagon · octagon · septagon · hexagon · pentagon · square · triangle

unstable · unstable · unstable · unstable · unstable · unstable · unstable · **stable**

How do multiple triangles interact?

Two triangles share a vector; two triangles with common side will flap about like a hinge.

Three triangles (each sharing two sides) will lock together as a stable structure.

But in the process, a fourth triangle is formed out of their unshared sides. (Sounds a bit like "Quaternary rests in Ternary" doesn't it?)

We have formed a tetrahedron. (In Greek, "tetra" means "4" and "hedron" means "sides")

Bucky calls the tetrahedron a "Tee-pee tripod" The legs of a 3-legged tripod might splay outwards under weight, but if the 3 legs are interconnected, such splaying is impossible.

He calls the tetrahedron, with its 4 triangular faces, the "**simplest structure**," the "**first and simplest subdivision of Universe.**" While the Greeks counted the sides of polyhedra, Bucky preferred to count the vectors (in this case, edges), declaring, "**Six vectors are required for complete multidimensional stability.**"
(Synergetics 1, Fig 621.10 p. 339)

One vector makes one edge.
Two vectors make two legs that are flexible like a geometer's compass.
Three vectors make the stable triangle, but we are still only in 2 dimensions.
Four vectors make a triangle with one shaky leg.
Five vectors make 2 triangles that are hinged.
But, *Six* vectors form 4 triangles that interlock as the most economical, stable 3-D shape possible.

The Greeks knew that there were only 5 "regular polyhedra" (many-sided figures with equal sides and equal angles).

tetrahedron	octahedron	icosohedron	cube	dodecahedron
4 triangles	8 triangles	20 triangles	6 squares	12 pentagons
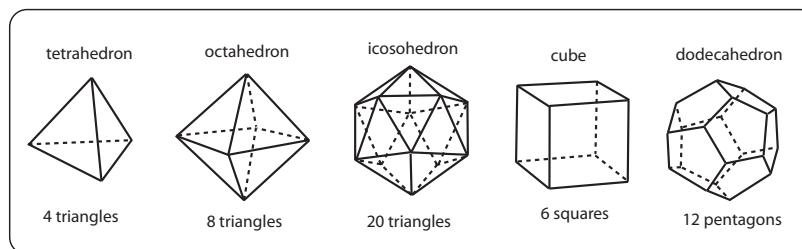
In *Timaeus 55-56*, Plato associates these "Platonic solids" with the Elements.

Plato might have based his associations on tactile feelings.

Plato's depiction of the regular polyhedra as the 4 Elements

"Fire"   "Air"   "Water"   "Earth"   "used to decorate the heavens"
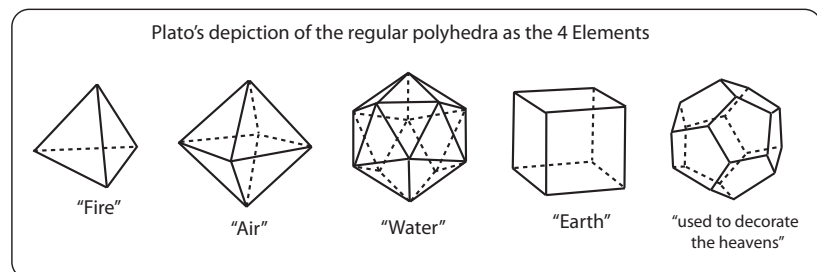
The roundest of these shapes, the icosahedron, flows smoothly.like water. The least spherical of these shapes is the cube, more in line with "earth" which clumps together. Plato calls the cube the "most immobile of the shapes." The tetrahedron has the sharpest points, the way fire can feel stabbingly hot when you get too close to it. The octahedron is an "intermediate" shape between sharp fire and flowing water and he associates it with air.

Plato didn't describe the fifth figure, which most historians assume to be the fifth regular polyhedron, the dodecahedron. All he says is that "the god used it for embroidering the constellations of the whole heaven."

Plato refers to the tetrahedron as *puramidos* or pyramid. The Greek word for fire is "pur," from which we derive modern words like pyrotechnic, pyromaniac, or funeral pyre. When Dee makes references to **pyrologists**, it might seems as though he's referring to alchemists who work with fire to transform matter, but he's really referring to geometers who study tetrahedra!

Nowadays, the word pyramid brings to mind the 4-sided Egyptian or Mayan pyramids. These are not tetrahedral because their bases are square, not triangular. (Technically a pyramid can also be 5-sided, 6-sided, or have any number of sides, as long as the sides are all equal triangles they meet at a common vertex.)

A 4-sided "pyramid of Giza" is actually one half of an octahedron. Each of its 4 faces is an identical equilateral triangle. Thinking in terms of vectors, the four "legs" are somewhat solidified by the square base.

But because of that square base the "4 sided plus base pyramid is not as stable as the "3 sided plus base" tetrahedron. Nonetheless, these two shapes do an incredible tango together!
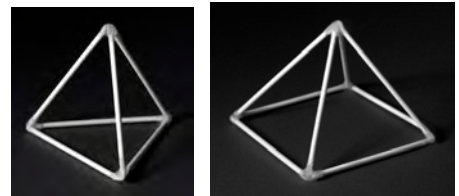
Describing this dance in words or drawings is challenging, so here I've photographed lollipop sticks which are hot glued together. (The subtle shadows and limited depth of field provide better visual clues about the interplay of these 3-dimensional shapes.)
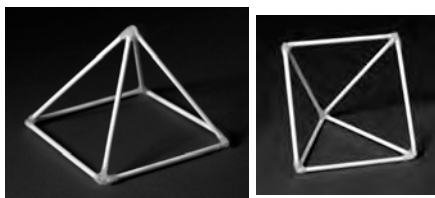
For example, here is a 4-sided pyramid on its base. And also lying on one of its triangular sides.
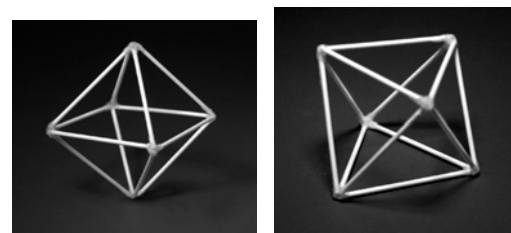
Here is the octahedron balanced on its tip. And also "tipped over," lying on one of its triangular faces.

Let's start with two tetrahedra
whose rear edges are aligned.
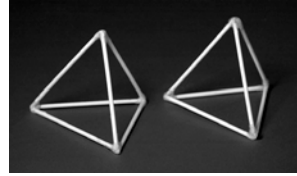
Next we'll slide them together so they touch,
then connect their 2 top points and 2 front points
together with lollypop-stick vectors.
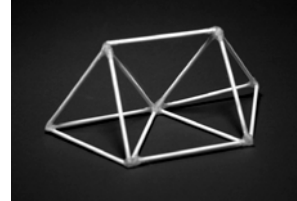
The "cavity" between the two tetrahedra is a
"4-sided pyramid of Giza" lying on its side.
(Here are the 3 shapes separated,
but remember they share edges,
so some edges have been duplicated)
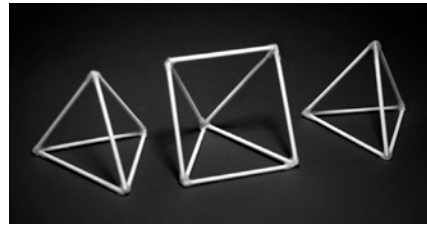
Next let's assemble 3 tetrahedra.

Let's stick them together
(so they form an equilateral triangle in their midst).

When the 3 top points are connected with lollipop sticks,
the cavity in the middle is an octahedron lying on its side.
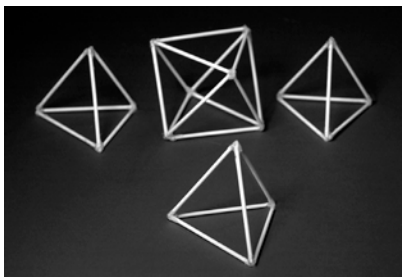
Here are two more views of this,
but remember,
this central octahedron
and the 3 tetrahedra
actually **share** edges.

Next, let's add a fourth tetrahedron
to the top of this assembly.
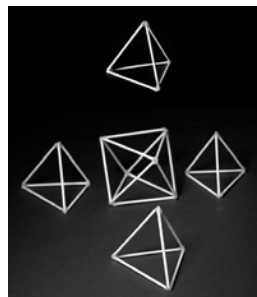
Suddenly we have a larger version of Nature's
most stable shape, **a giant tetrahedron**.
(Note that all the tips of the inner octahedron
are at the midpoints of the edges
of the giant tetrahedron.)

Even though the center of this giant tetrahedron is an octagon, its essence is still "four-ness." The octahedron is essentially a 4-sided square with 4 legs on top and 4 legs on the bottom. It's a sandwich of two 4-sided pyramids of Giza. In ye olden days it might be called "**Quaternary**."

Likewise, even though there are 4 tetrahedra at the corners of this giant tetrahedron, each of these mini-tetrahedra have, at their essence, three-ness. Each of them can be seen as 3 legs resting on a 3-sided base, plus all of its faces and bases are equilateral triangles. In ye olden days this might be called "**Ternary**."

In this sense, this "giant tetrahedron" seems to be expressing "**The Quaternary rests in the Ternary!**" The "Quaternary" octagon is nestled comfortably in a nest of "Ternary" tetrahedra. They fit perfectly together like a tenon fits a mortise, like yin fits yang, like a fastball fits a catcher's mitt. They were made for each other.

Boiling all this down to its essence,
the tetrahedron and
the "half-octahedron"
dance this way.

The tetrahedon
and the "full octahedron"
boogie like this:

This is the essence of the shape that young Bucky intuitively
constructed out of dried peas and toothpicks in kindergarten.
It's also the shape for which Bucky was awarded
US Patent 2,986,241 in 1961.
It has subsequently become
a common shape in building construction.
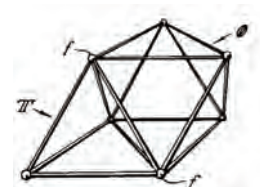
Perhaps a row of these alternating tetrahedra and "tipped-over Pyramids of Giza" will look more recognizable.

This stable shape is used for cranes, boom-arms, TV and radio station antennas, portable stages, and to hold up billboards and highway signs.

Because it so amazingly rigid and lightweight, it's used as beams to span large interior spaces like factories and "box stores."

A series of connected rows make an amazingly rigid floor platform or roof. Bucky named this structure the "**octet truss**."

("Oct" is short for octahedra, and "tet" is short for tetrahedra, and the "truss" means "framework.")

It has become so popular that contractors simply call it "space frame" or "space structure." Even the US government specifies the "octet truss" in its bids for the main structuring of space stations.
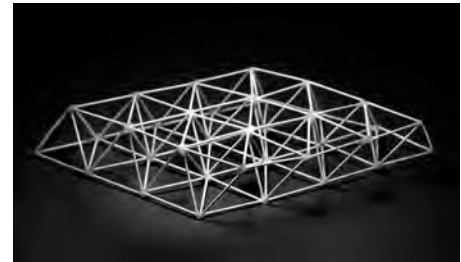
Though Bucky holds the patent for this structure, it was actually used by the great inventor Dr. Alexander Graham Bell in the early 1900's. He used it to build a tall antenna and also in a kite large enough to carry a man.
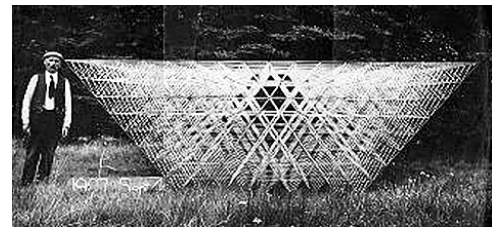
When several layers of octet trusses
are stacked together, it's obvious
how they can be considered "all-space filling."
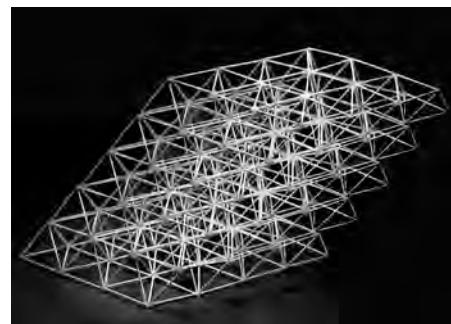
All these intersecting and overlapping lines
get very confusing,
so let's just look at two layers of the octet truss.
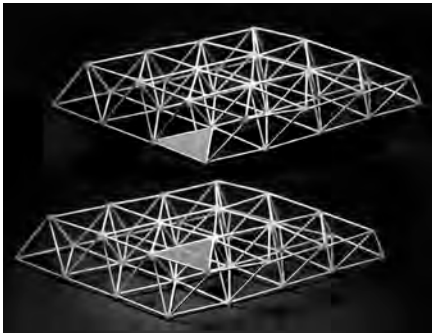
I've highlighted (in grey)
the two triangles which mate together.

Note that the tetrahedra of the top layer
are not situated exactly above
tetrahedra of the lower level.

The base of the front corner tetrahedron
of the upper layer actually aligns
with the triangular face of a
"tipped over" octahedron of the lower layer.
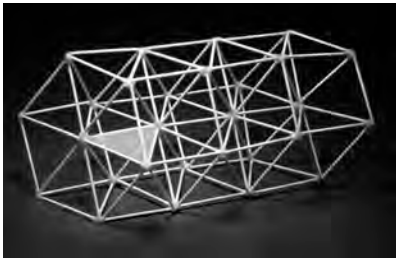
Here the two layers are combined,
showing where the two grey triangles meet.

(To make it less confusing,
I've eliminated some of
the back and front of the structure.)
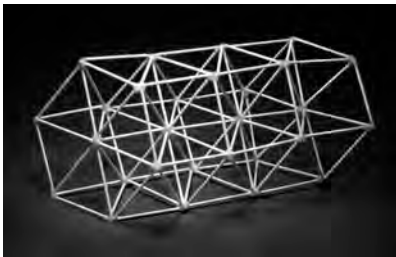
Look at the "hubs" of the vectors
deep in the midst of the arrangement.

(You can see two of them here.)

Each "hub" has 12 vectors radiating outwards.
Let's eliminate everything
except one of these intersection points,
its 12 radiating vectors
and the vectors that connect their outer tips.

Even when totally isolated,
this shape is little hard to discern.

But when it's rotated slightly,
it's apparent that it is a **cuboctahedron,**
one of the 13 "semi-regular" polyhedra
described by Archimedes around 200 BC.
(Though Heron of Alexandria claims Plato
knew about it as well, around 325 BC.)

"Semi-regular" means it has 2 or more different shaped faces
(as opposed to each of the regular Platonic solids,
whose faces are made from one kind of shape.)

The cuboctahedron has 8 triangular faces and 6 square faces.
Each of the 12 radiating vectors and the 24 edge vectors
is exactly the same length!

This might not seem like an unusually special charateristic, but it really is!
Here is a brief description of the 5 Platonic regular polyhedra
and the 13 Archimedean semi-regular polyhedra.

	types of faces		edges	radiating vertices	total faces
5 "Platonic Solids"	4 triangles	tetrahedron	6	4	4
	8 triangles	octahedron	12	6	8
	20 triangles	icosahedron	30	12	20
	6 squares	cube	12	8	6
	12 pentagons	dodecahedron	30	20	12
13 "Archimedean Solids"	**8 triangles and 6 squares**	**cuboctahedron**	**24**	**12**	**14**
	20 triangles and 12 pentagons	icosidodecahedron	60	30	32
	4 triangles and 4 hexagons	truncated tetrahedron	18	12	8
	8 triangles and 6 octagons	truncated cuboctahedron	36	24	14
	6 squares and 8 hexagons	truncated octahedron	36	24	14
	20 triangles and 12 decagons	truncated dodecahedron	90	60	32
	12 pentagons and 20 hexagons	truncated icosahedron	90	60	32
	8 triangles and 18 squares	rhombicuboctahedron	48	24	26
	12 squares and 8 hexagons and 6 octagons	great rhombicuboctahedron	72	48	26
	20 triangles and 30 squares and 12 pentagons	rhombicosidodecahedron	120	60	62
	30 squares and 20 hexagons and 12 decagons	great rhombicosidodecahedron	180	120	62
	32 triangles and 6 squares	snub cube	60	24	38
	80 triangles and 12 pentagons	snub dodecahedron	150	60	92

Only the cuboctahedron has radiating vectors which are the same length as its edge vectors.
On this chart it scores a perfect "1.0000000000"

length of edge vector
length of radiating vector

1.6329931619	tetrahedron	
1.4142135624	octahedron	**5 "Platonic Solids"**
1.0514622242	icosahedron	
1.1547005384	cube	
0.7136441795	dodecahedron	
1.0000000000	**cuboctahedron**	
0.6180339887	icosidodecahedron	
0.8528028654	truncated tetrahedron	
0.5621692754	truncated cuboctahedron	
0.6324555320	truncated octahedron	**13 "Archimedean Solids"**
0.3367628118	truncated dodecahedron	
0.4035482123	truncated icosahedron	
0.7148134887	rhombicuboctahedron	
0.4314788105	great rhombicuboctahedron	
0.4478379596	rhombicosidodecahedron	
0.2629921751	great rhombicosidodecahedron	
0.7442063312	snub cube	
0.4638568806	snub dodecahedron	

(adapted from David Sutton, p. 36-37 and 56-7, Platonic and Archimedean Solids,
NY, Wooden Books, Walker and Co., 2002)

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Bucky views a vector as a line segment connecting two special points: the center points of the two tangent spheres.

This line segment goes through what Bucky calls the "kissing" point of two spheres. Thus, the radius of each of these spheres is half the length of the vector. If each of these two radii is given the value of "1", then the whole vector has a value of "2." Bucky calls very simple relationships "a prime concept of synergetics vectorial geometry" and succinctly summarizes it this way:

> "Unity is plural,
> and at minimum two."
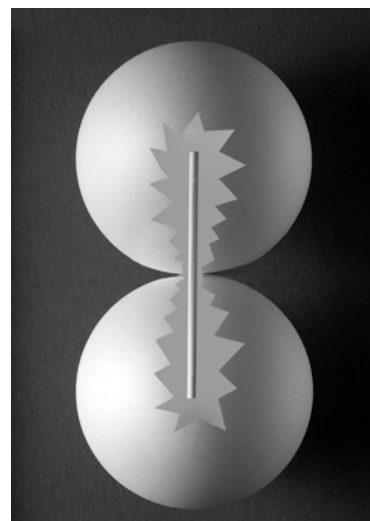
Bucky sees vectors not simply as lollipop sticks or even as line segments, but as an "energy relationship," and relationship requires at least two entities. As Bucky puts it, "the word *unity* means union, which is inherently at minimum two." He simplifies his maxim even more by simply writing:

> Unity=2

A catchier popular expression comes from a 1952 song: "It takes two to tango."

John Dee saw this same thing, only he expressed it differently, **"A *Unit* is that Mathematical thing, Indivisible by participation of some likeliness of whose property any thing which indeed or is counted One, may reasonably be called One. We account a *Unit*, a thing *Mathematical* though it be no *Number* and also indivisible."**

Thus, the first divisible thing must be 2. Dee adds in the margin *"Note the word, Unit, to express the Greek Monas & not Unity, as we have all, commonly, till now used."*

(Dee, *Preface to Euclid*, p.j):

It doesn't take much imagination to see how Dee cryptically refers to two spheres in the Monas Hieroglyphica. They are the Sun and the Moon.

In the Monas symbol, the Sun is portrayed as a circle, the Moon is portrayed as a half-circle, and they overlap. But the real Sun and Moon are both spheres (that appear to the eye to be the same size) and they don't actually overlap (o.k., except during eclipses). In the Monas symbol, the radius of the Moon's half-circle and the Sun's circle are exactly the same. Also in Theorem 4 Dee notes that when the Moon is full, she "becomes transformed" into the Sun.

In the emblem following Theorem 24, Dee uses the expression "Contactus ad Punctum" or "Contact at a point." Even though he illustrates at the tangency of a line and a circle, the tangency of two circles is not that different. "**Kissing point**" and **"Contactus ad Punctum"** are the same thing. Bucky and Dee are on the same wavelength, they just used different terminology!

Consideration 5: Closest Packing of Spheres

In this cut-away view of 3 tangent spheres,
3 vectors are required to connect the 3 centerpoints.
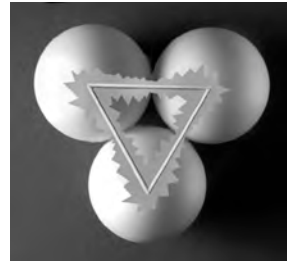
And here, 4 vectors connect
the 4 centerpoints of 4 spheres arranged in a square.
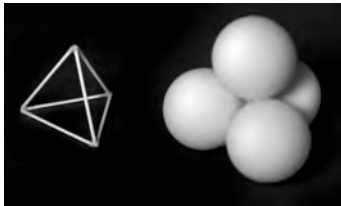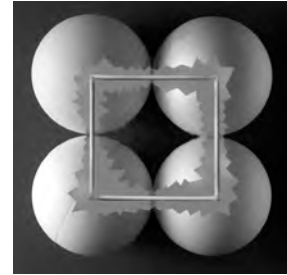
Thus, a tetrahedron of spheres consists of
3 tangent spheres
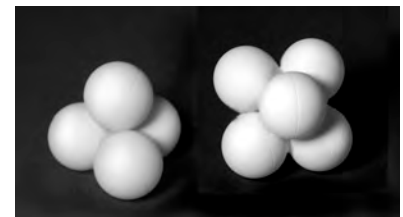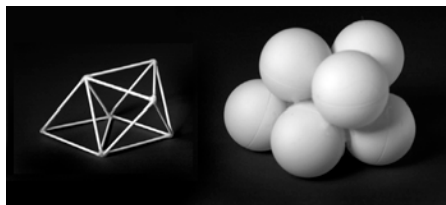plus fourth sphere sitting in the nest they create.
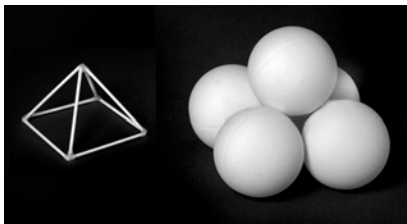
In a half-octahedron,
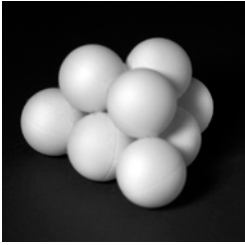one sphere sits in the nest formed by 4 others.

In this depiction of
a tetrahedron and a half-oc-
tahedron lying on its trian-
gular face, note how these
shapes "share" two spheres.
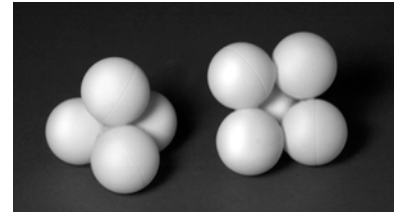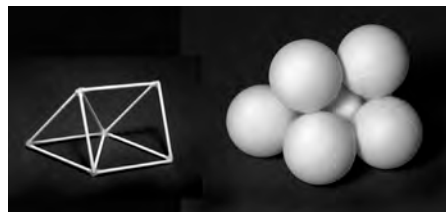
Here is the essence
of Bucky's "octet truss,"
the octahedron-tetrahedron
tango,

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Here's that configuration of 3 tetrahedra
and the "tilted octahedron" that rests in their midst.

The 3 tetrahedra not only share
spheres among themselves,
but the octahedron
is totally occupied by spheres
which are also parts of the 3 tetrahedra.
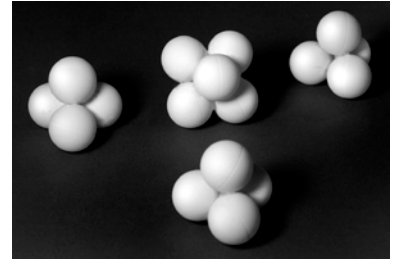
When a fourth tetrahedra is added on top of this assemblage
we have the "giant tetrahedron."
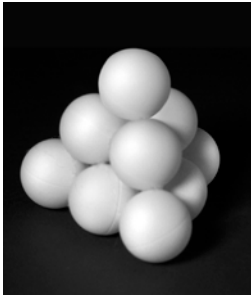
Bucky calls this the
"optimally most stable and efficient
aggregating arrangement
known for past centuries
by stackers of unit-radius
coconuts or cannonballs."

(Fuller, Synergetics 2, p. 239)
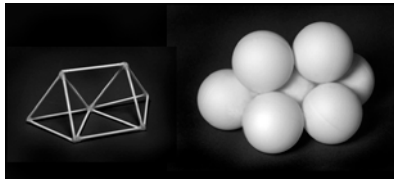
Here's how a row
of an octet truss starts out.
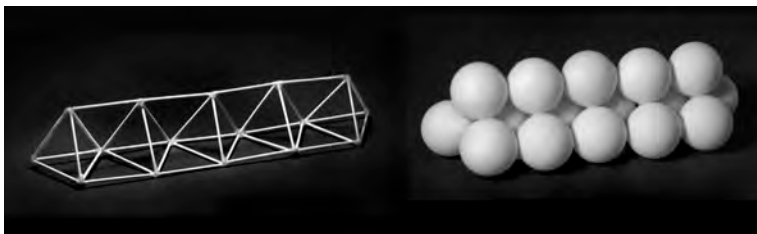
And here is that same pattern
extended a little ways,
making a
"TV station antenna"
of spheres.
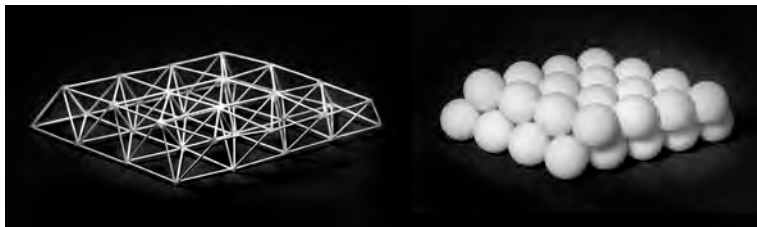
We can extend that pattern to make
a layer of "space frame" of spheres.

Note that there is one closely-packed grid
of spheres on top of another.

But they are not **directly** on top;
they are "shifted" so they all fit
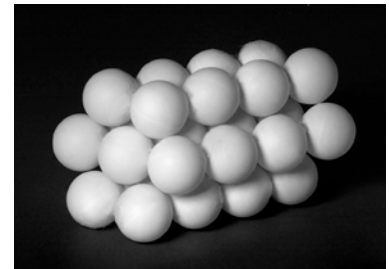comfortably in nests
created by the grid of spheres
below them.
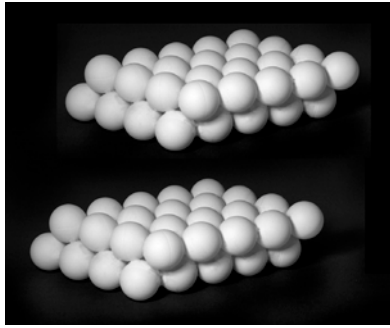
To make two layers
of space frame,
one might think
2 more layers must be added.
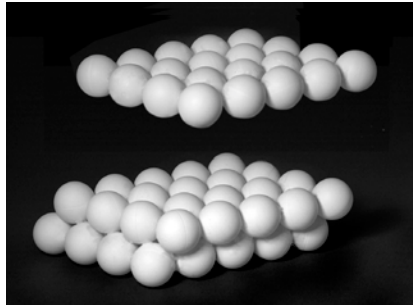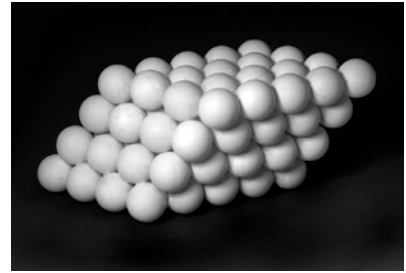
However, as spheres
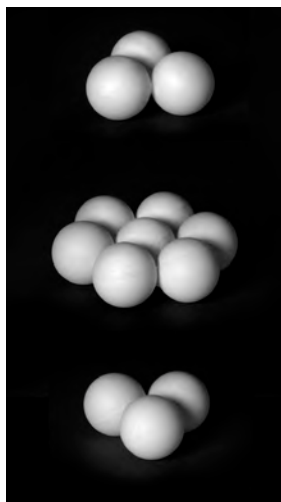are "shared,
actually only one
additional layer
is required
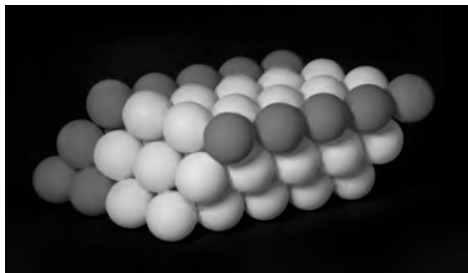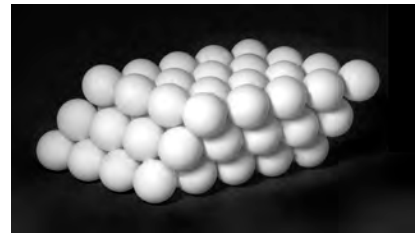
Two space grids
on top of each other
start to get a little
visually overwhelming,
so let's eliminate some
for the sake of seeing
what's going on here.
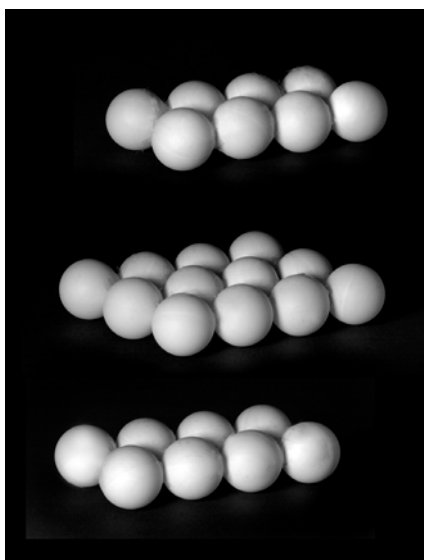
Actually, let's take it
a step further
and separate this
assembly into layers.
Now, we can see one
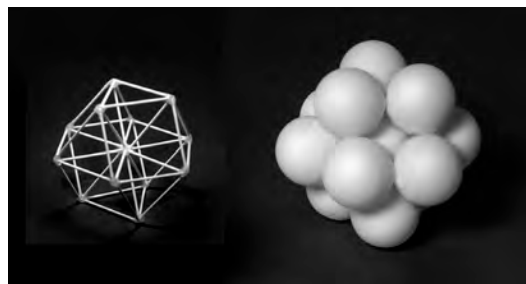of those
"nucleus" spheres
and all its surrounding
"tangent spheres."
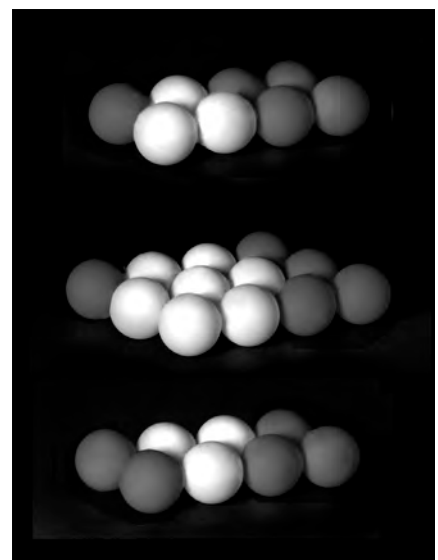
A hexagonally shaped
"6-around-1"
grouping of spheres
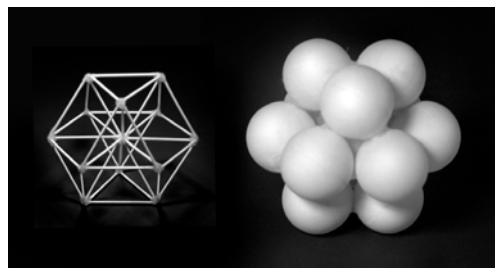is sandwiched between
two trianglular groupings.

When reassembled,
it becomes
a little clearer
that this is a
cuboctahedral shape.

This view accentuates
a square face of 4 spheres
like the "square"
of lollipop sticks.

The cuboctahedral shape
is a bit more recognizable
if the whole thing is rotated a bit
to accentuate a triangular face.

To summarize,
the octet truss of vectors and
the closest packing of spheres arrangement
are essentially the same thing!

All of the spheres buried deep within the multiple layers of an "octet truss of spheres" is surrounded by exactly 12 spheres.

This total of 13 make a cuboctahedral conglomeration, which "share" spheres with neighboring cuboctahedral conglomerations.

Consideration 7: Vector Equilibrium

Bucky calls this cuboctahedral shape a "vector equilibrium," as **all the radiating vectors and the edge vectors are equal**. They reinforce each other with triangular stability making a shape with overall "**equilibrium**."

(from the Latin word *aequilibrium*; *aequi* meaning "equal" and *libra* meaning "balance.")

Bucky even invented a symbol for a "vector equilibrium."

Bucky calls the octet truss or (the space frame) the "**isotropic vector matrix**." Isotropic means "everywhere the same," "matrix" means an array or organizational structure.

(from the Latin *matricibus* meaning "womb," which comes from *mater* meaning "mother.")

While Bucky's terms "isotropic vector matrix" and "vector equilibrium" are brilliantly descriptive descriptive, I will most often use the simpler terms "octet truss" and "cuboctahedron."

All these terms essentially are describing the same arrangement:

alternating tetrahedra and octahedra,
isotropic vector matrix,
octet truss,
space frame,
space structure,
12 radiating vertices surrounding 1,
vector equilibrium,
cuboctahedron,
only shape in which "vectorial lengths and interanglings are everywhere the same"

(last description by Fuller, *Synergetics* 1, section 986.140, p.238)

Consideration 8: Concentric Polyhedral Hierarchy

Volumes

Bucky discovered that the 5 Platonic polyhedra and vector equilibrium are interrelated in various ways. For example, if a tetrahedron has a volume of 1, other polyhedra having the same edge lengths will have the following volumes:

Tetrahedron 1
Octahedra 4
Cube 3
Vector Equilibrium 20
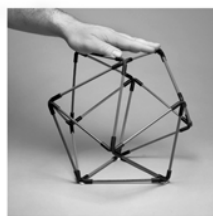
Bucky's Jitterbug

Another example of this interrelationship is what Bucky calls the "Jitterbug" Transformation. To demonstrate, he made a model of the edges of vector equilibrium by joining dowels with flexible rubber. (A preassembled toy that demonstrates this principle called "Vector Flexor" can be purchased at the Buckminster Fuller website.)

Place one triangular face of a Vector Flexor downwards on a flat tabletop. With the palm of your hand, push down on the opposing top triangular face. The whole assembly contracts symmetrically. First it morphs into an icosahedron, then into an octahedron. Let's take it a step at a time.

Bucky's "jitterbug" transformation

from vector equilibrium…   …to icosahedron…   …to octahedron

icosahedron...

During the firsr transformation,
the triangular faces hold their shape (of course).

But the square faces collapse
into diamond shapes (rhombuses).

A square face collapse
into diamond shape

Add an edge length,
and the diamond becomes
2 equilateral triangles

If an additional edge length (lollipop stick)
is put ascross the narrow width of the diamond shape,
it becomes 2 equilateral triangles.

If this is done to each of the 6 diamonds
(made from the original 6 square faces),
12 equilateral triangles have been created.
These 12, plus the original 8 triangular faces
of the vector equilibrium, make a total of 20,
the number of triangular faces of an icosahedron.

octahedron

Let's not add them, but instead keep pushing downwards.
The diamond faces get skinnier and skinnier,
until they finally vanish.
Now the whole assembly has morphed
into an octahedron (with its 8 triangular faces.)

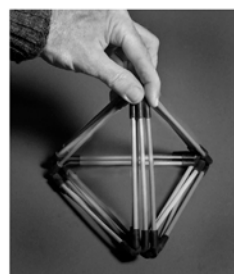There's are even more steps that can be considered as part of this dance.
If you release the downward pressure and twist,
the whole assembly can be flattened
into a triangular grid comprised of 4 smaller triangles.
Fold the 3 outer triangles up so their tips meet and presto!
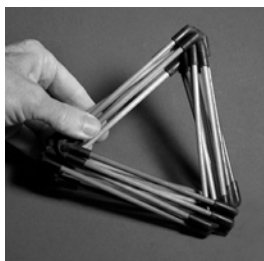You've got a tetrahedron.

...flattened into
4 triangles...
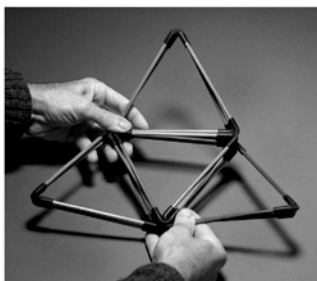
... folded up so that
the tips meet...

...forming a
tetrahedron

The tetrahedron
folds down
into a triangle.

...back to a
vector
equilibrium...

But there's more!
Fold the sides of the tetrahedron down flat,
and suddenly you have a triangle.
Actually, its a thick stack
of the original 8 triangular faces.
The 6 original square faces
have all disappeared.

The dance is done.
Let go,
and the model springs back outwards
and becomes a vector equilibrium again.
The square faces even reappear.

This "jitterbugging" shows
that the cuboctahedron
is related to the icosahedron,
octahedron, and tetrahedron
(all of which have triangular faces).

JITTERBUG TRANSFORMATION

Vector equilibrium
or cuboctahedron

Icosahedron

Octahedron

Tetrahedron

Duals

More insight into the the "hierarchy" of poyhedra cas be gleaned by studying "duals." and "intersections." These ideas are related, but let's start with duals.

If the vertices of one polyhedra correspond to the face centerpoints of another polyhedron, they are duals.

The easiest duals to envision
are the 6-sided cube
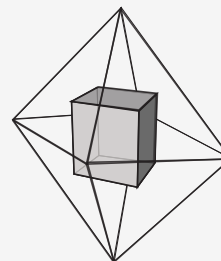and the 8-sided octahedron.

The 6 corner points of a small octahedron
coincide with the
6 face centerpoints of a larger cube.

The 8 corner points of a small cube
coincide with the
8 face centerpoints of a larger octahedron.

The cube and the octahedron are "duals".
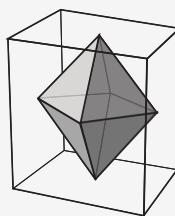
The corner points
of a small octahedron
are the
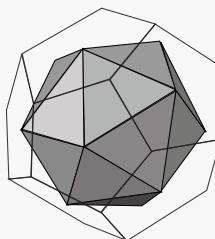face centerpoints
of a larger cube.

The corner points
of a small cube
are the
face centerpoints
of a larger octahedron.

Likewise, the 20-sided icosahedron
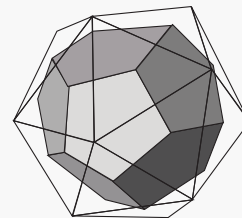and the 12-sided dodecahedron are duals.

The 12 corner points of a small icosahedron
coincide with the
12 face centerpoints of a larger dodecahedron.

The 20 corner points of a small dodecahedron
coincide with the
20 face centerpoints of a larger icosahedron.
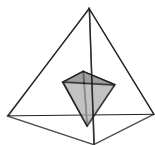
The dodecahedron and the icosahedron are "duals".

The corner points
of a small icosahedron
are the
face centerpoints
of a larger dodecahedron.

The corner points
of a small dodecahedron
are the
face centerpoints
of a larger icosahedron.

The tetrahedron
is a "self dual."

The corner points
of a small tetrahedron
are the
face centerpoints
of a larger tetrahedron.

The tetrahedron is the only "self-dual."
The 4 vertices of a small, upside-down tetrahedron
coincide with the
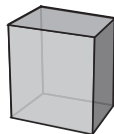4 face centerpoints of a larger, upright tetrahedron.

(Incidentally,
the dual of the cuboctahedron itself
is a rhombic dodecahedron,
which has 12 diamond-shaped faces.)

Intersections

The intersection of a 6-sided cube
and an 8-sided octahedron
is a 14-sided cuboctahedron,
Bucky's vector equilibrium.
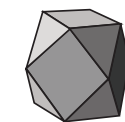
The
intersection...

...of a
cube...

...and an
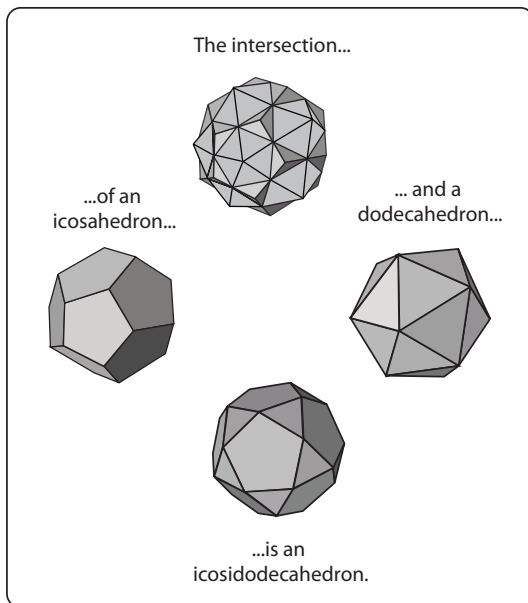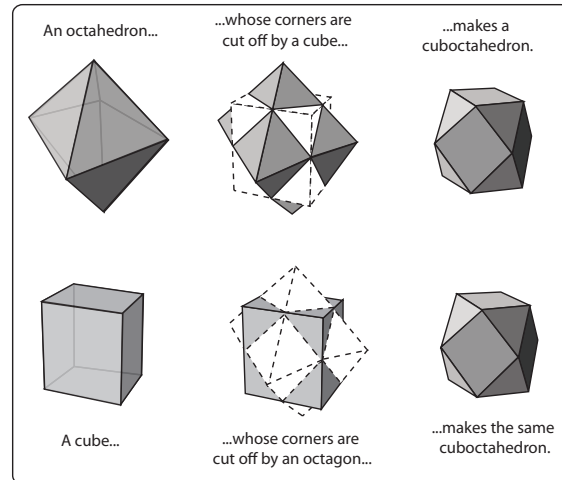octahedron...

...is a
cuboctahedron.

There are two ways to look at this:

First, as a cube,
used to "slice off"
the 6 pointy corners of an octahedron.

Or second, as an octahedron,
used to "slice off"
the 8 pointy corners of a cube.

In each instance, the slicing is done
at the midpoints of the edges.
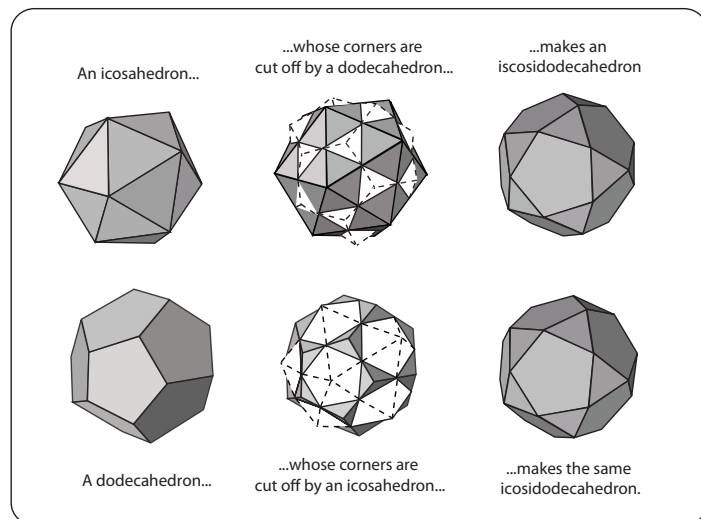(And the resulting cuboctahedrons are identical.)

An octahedron...

...whose corners are
cut off by a cube...

...makes a
cuboctahedron.

A cube...

...whose corners are
cut off by an octagon...

...makes the same
cuboctahedron.

The intersection...

...of an
icosahedron...

... and a
dodecahedron...

...is an
icosidodecahedron.

The intersection of a 20-sided icosahedron
and a 12-sided dodecahedron
is a 32-sided isocidodecahedron.

This Archimedean solid has
12 pentagonal faces
and 20 triangular faces.

There are two ways to look at this.

First, as a dodecahedron,
used to "slice off"
the 20 pointy corners of an icosahedron.

Second, as an icosahedron,
used to "slice off"
the 20 pointy corners of a dodecahedron.

An icosahedron...

...whose corners are
cut off by a dodecahedron...

...makes an
iscosidodecahedron

A dodecahedron...

...whose corners are
cut off by an icosahedron...

...makes the same
icosidodecahedron.

As the tetrahedron is a "self dual,"
guess what the intersection
of an upright and an inverted tetrahedron is?

It's an octahedron.

(lying on one of its triangular faces, not on its tip).

The intersection...

...of a
tetrahedron...

...and another
tetrahedron ...

...is a small
octahedron.

A tetrahedron...

...whose corners are cut off
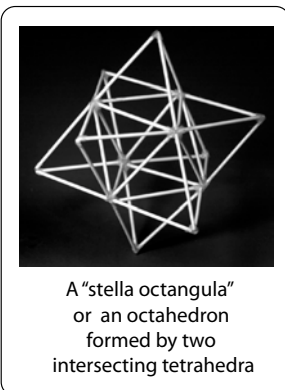by another tetrahedron...

...makes an octahedron.
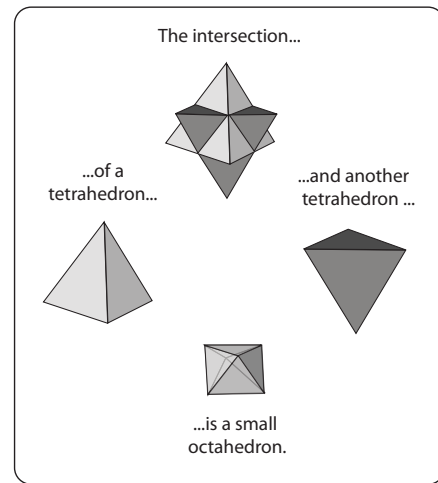
Envision the inverted tetrahedron slicing off
the 4 pointy corners of the upright tetrahedron.
What remains is an octahedron.
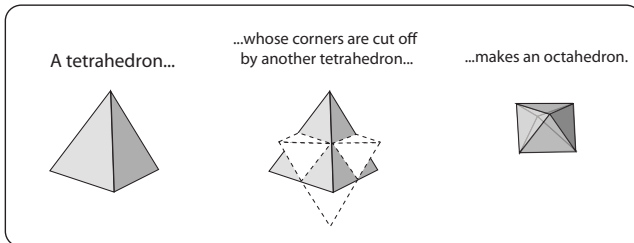
This is the same octahedron
we saw in Bucky's octet truss.
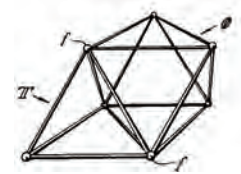
It's is the same octahedron we saw previously,
in the heart of the "giant tetrahedron"
of lollipop sticks.

Geometers call this combination
of two tetrahedra a "Star Tetrhedron."
It's kind of lke a 3-D Star of David.

Each of the 8 triangular faces of the internal octahedron
is "shared"
by one face from each of the 8 surrounding tetrahedra.

A "stella octangula"
or an octahedron
formed by two
intersecting tetrahedra

In his 1509 *Divina Proportione*,
Luca Pacioli calls this shape "stella octangula,"
meaning "stellated or star octahedron."
(Pacioli's illustrations were done by by Leonardo da Vinci)

The main theme I'm driving at here in this study of "Volumes, Jitterbugging, Duals, and Intersections" is this: Even though the cuboctahedron is an Archimedean polyhedron (with 2 different kinds of faces), it's still **very** interrelated with those 5 Platonic polyhedra (which only have one kind of face).

Let's bring the sphere, that wonder of Nature, into the picture.

Great circles and the Cuboctahedron

The "fiery" tetrahedron is the "sharpest" or pointiest of the Platonic polyhedra because it has the fewest number of sides, that is, 4. The cube has 6 sides, the octahedron has 8, the dodecahedron has 12, and the icosahedron has the most with 20.

When the 20 triangular faces of the icosahedron are subdivided into smaller triangles, it makes the geodesic sphere. Break those smaller triangles into even smaller ones and the geodesic sphere become even rounder.

It never becomes perfectly spherical, but if you look at Bucky's dome "Spaceship Earth" at Epcot in Disney World, it certainly looks like a sphere. It has 954 triangular panels and is 165 feet in diameter, (that's over half a football field).

Just as *triangle…square…pentagon…hexagon…* progress towards circularity in the 2-D realm, *tetrahedron…cube…octahedron…dodecahedron...icosahedron…*progress towards sphericity in the 3-D realm.

A useful way to see the relationship between the various polyhedra and the sphere is with "spherical polyhedra." This term sounds paradoxical, but it simply means is projecting the edges of a polyhedron outward into a sphere.

A regular
cuboctahedron…

…blown up like a balloon
into a
"spherical cuboctahedron"
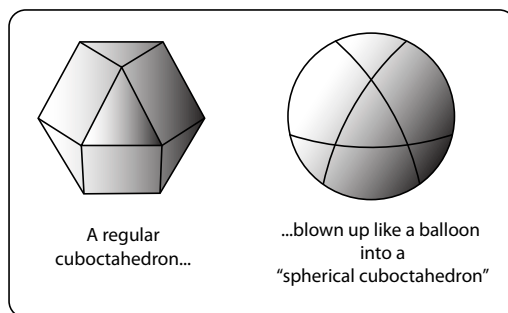
For example, imagine a cuboctahedron with a rubber skin that could be blown up into a spherical balloon.
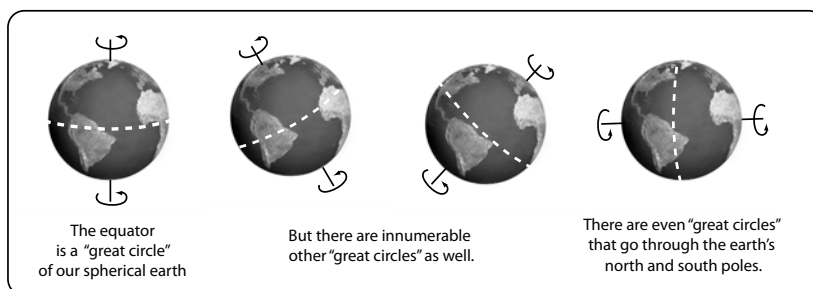
The straight edges will curve to conform to the curvature of the sphere, but the 12 vertices will remain in their same places.

Next, imagine sticking a thin pole through one of those 12 vertices (this conceptual balloon doesn't pop), then through the centerpoint of the sphere and then out the opposite side at another vertex point.

Holding the top and bottom of this pole so its vertical, spin the white sphere along a freshly painted black wall.

A line will be drawn along the "equator" that goes around the whole sphere. This is called a "great circle" (as it's the "greatest-sized" circle that can be drawn on a sphere).

The great circle for the "north pole and south pole" axis of the earth is the Equator. But any number of poles can be made, each making distinctly different great circles. Bucky calls each of these poles a "pole of spinnability."

The equator
is a "great circle"
of our spherical earth

But there are innumerable
other "great circles" as well.

There are even "great circles"
that go through the earth's
north and south poles.

As the cuboctahedron has 12 **radiating vectors**, it has 6 "poles of spinnability." Thus we might run it along that black wall a total of 6 different ways, creating 6 different great circles.

But there are still more "poles of spinnability. Imagine a pole going through one triangular **face**, through the sphere's centerpoint, and out the opposite triangular face. The same thing could be done through opposing square faces.

As the cuboctahedron has 14 faces, there are 7 such poles, each of which will make 7 more distinctively different great circles. (Let's run it along the black wall 7 more times)

But wait, there's more. Imagine running a pole through the midpoint of any of the **edges** (which are now curved, conforming to the sphere), through the sphere's centerpoint, and out through the midpoint of the edge on the opposite side. As these are 24 edges, 12 such poles can be made, making 12 more great circles if run along the black wall.
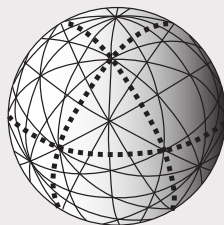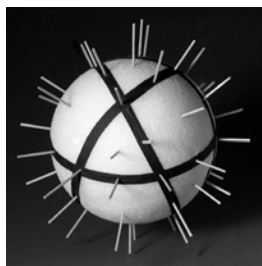

Great circles of a cuboctahedron

12 radiating vectors make 6 poles or 6 great circles
14 face centerpoints make 7 poles or 7 great circles
24 edge midpoints make 12 poles or 12 great circles
_____
25 great circles in total

Let's tally up the great circles:
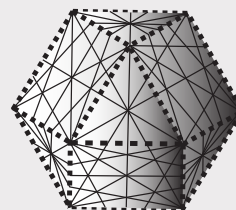
(In this accounting, notice that the 6 radiating vector poles, plus the 7 face-centerpoint poles, total to 13 poles.

These 13 poles, plus 12 edge-midpoint poles, equals 25.)

The 25
"poles of spinnability"
of a spherical
cuboctahedron

The 25 great circles
of a cuboctahedron
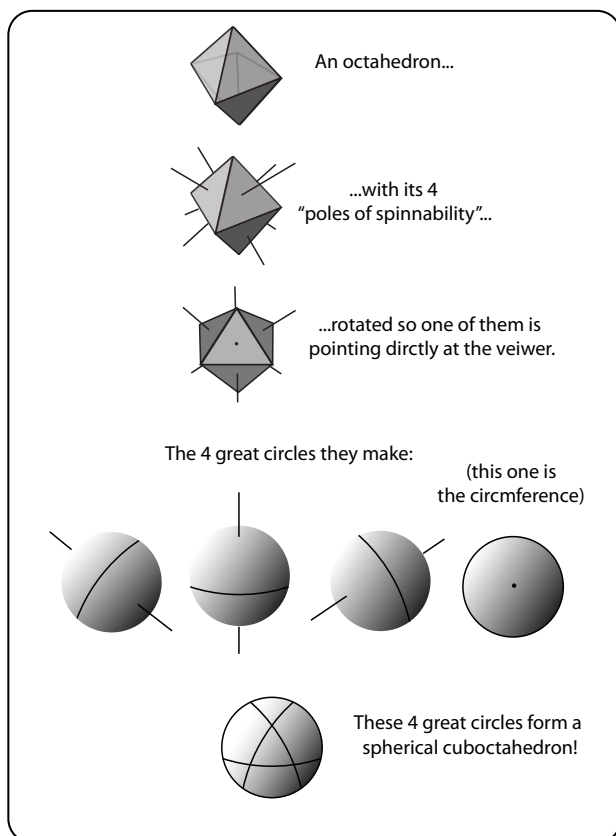which has been
"ballooned" into a sphere

Those 25 great circles
applied to the faces
of a cuboctahedron

You can see by this math that the number of great circles in the Platonic and Archimedean polyhedra will always be **one half** of the total number of radiating vertices, edges, and face centerpoints. (It takes two opposing points to tango as a pole.)

***Can you guess what shape is made by the great circles of the poles
which pass through the face centerpoints of an octahedron?***

The octahedron has 6 radiating vertices, plus 12 edges, plus 8 face centerpoints,
totaling 26 things.
This means there are 13 "poles of spinability" or 13 great circles.

An octahedron...

...with its 4
"poles of spinnability"...

...rotated so one of them is
pointing dirctly at the veiwer.

The 4 great circles they make:

(this one is
the circmference)

These 4 great circles form a
spherical cuboctahedron!

The 4 poles of spinnability **that go through the octahedron's 8 face centerpoints** must be particularly noted.

(You'll get dizzy trying to picture it in your mind, but these illustrations will make it easier to envision.)

Starting with a clear depiction of an octahedron, let's rotate it so one of the triangular side is facing towards us.

(One of the poles of spinability is aimed directly at us, so we actually see its great circle as a full circle.)

When these 4 great circles are combined, look what we have: a spherical cubotahedron!

Here, you can see 4 of its "triangular faces" and 3 of its "square faces. (The same pattern is inverted on the flip side of the sphere.)

Another interesting relationship is that
**the number of great circles will always
be one more than the number of edges.**

(after Edmundson, p. 230)

	Edges	Great Circles
Tetrahedron	6 + 1=	7
Octahedron	12+1=	13
Cube	12+1=	13
Cubuctahedron	24+1=	25
Icosahedron	30+1=	31

Octahedra and cubes each have the same number of faces (12),
but they are different shapes (triangles and squares, respectively.)
Still, each of these polyhedra has the same number of great circles (13).
Remember, the cubocthedron, is the intersection of a cube (12 edges)
and an octahedron (12 more edges).
These 24, plus 1, make for 25 great circles.
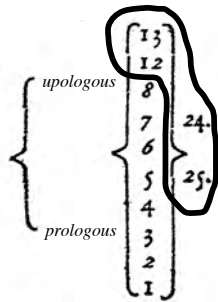
Does all this discussion of the numbers 12, 13, 24, 25, ring a bell?

These are the 4 numbers Dee included in the "Below half" of his Thus the World Was Created chart, and also in the Artificial Quaternary chart.

These numbers were some of the first clues that led me to suspect that Dee (in the mid-1500's) was as fascinated with the cuboctahedron as much as Bucky was (in the mid-1900's).
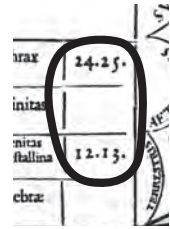
Consideration 9: Synergetics

Bucky explains that he had adopted the eight previous Considerations by the time of his 1927 life-altering moment on the shore of Lake Michigan. Each of the 8 considerations was "progressively amplified by subsequent experience-induced considerations" which he elucidates in his 1975 *Synergetics 1* and his 1979 *Synergetics 2*. (Remember, Bucky was 80 years old in 1975)
Just like his "unity is plural and at minimum two" theme, Bucky saw *Synergetics* as one book having two volumes.

Synergetics 2 is **not** a sequel to *Synergetics 1*, it's an extension of it. Both books have the **same** 12 chapter titles, **share** the same sub-chapter numbering system, and have **common** index.

Both Synergetics1 and Synergetics 2
have the same 12 chapters:

100 Synergy
200 Synergetics
300 Universe
400 System
500 Conceptuality
600 Structure
700 Tensegrity
800 Operational mathematics
900 Modelability
1000 Omnitopology
1100 Constant Zenith Projection
1200 Numerology

Consideration 10
The Spheric Experience:
Energetic-reality Accounting vs. Abstract-cubic Accounting

You can tell from this title that Bucky wasn't a big fan of the boxy Cartesian Coordinate systyem.
Energy radiates,
so he preferred using the Spherical Coordinate System.
This is the system astronomers use when they measure the azimuth, declination and distance of a star.

The
Spherical Coordinate
System

The "cubic-based"
Cartesian Coordinate System
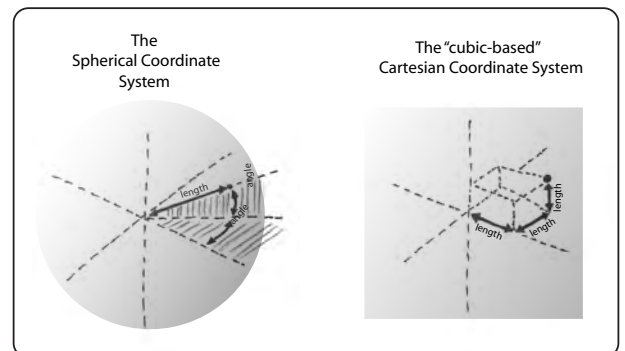
For a more comprehensive analysis, Bucky broke the tetrahedron into 24 equal-sized subdivisions called "A Quanta modules."

He found another subdivision of the Octahedron, which he called the "B Quanta module." He goes into great length explaining how these subdivisions relate to the other polyhedra and the octet truss. (I won't go into the details here.)

The Cuboctahedron and Closest Packing of Spheres
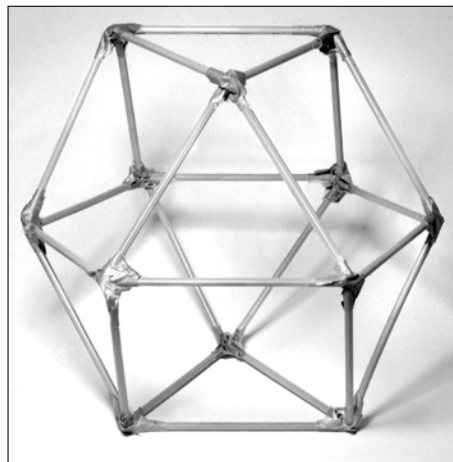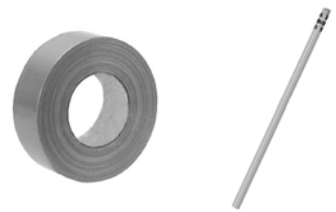

A cuboctahedron made from 24 pencils

### *Make your own cuboctahedron in 5 minutes or less*

A great way for you to get the feel of what's going on here
is to make your very own cuboctahedron in 3 easy steps.
It's inexpensive, fast, fun and fruitful.
Needed: 24 new, unsharpened pencils and duct tape

Step 1: Duct tape 4 pencils together to form a square,
then make 5 more squares this same way:
(Don't worry if the tape job is sloppy or overdone, as long as it holds.)

Make 6 squares with the pencils
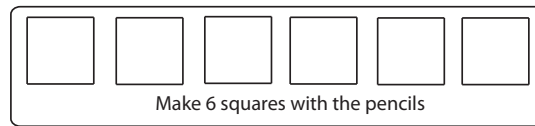
Step 2: Tape 3 squares together in this pattern:
(Remember to keep one central triangle pointing upwards
and the other pointing downwards.)

Tape the squares together in these two arrangements
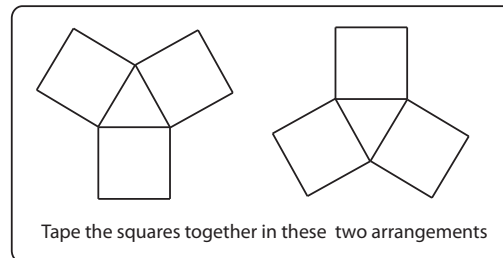
Step 3: Tape these two arrangements together
at their outer corners and you're done!

Tape the two arrangements together

That's all there is to it!

Another method is to make 8 triangles
and tape these two arrangements together.
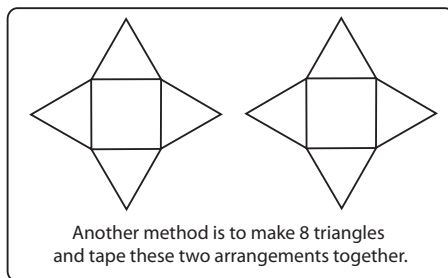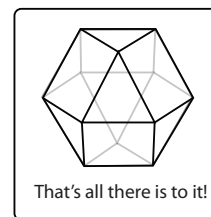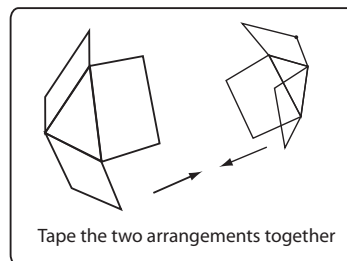
An alternative method is to make the 8 triangles,
and join them this way.
(This method is actually more poignant,
as it accentuates the triangular faces which, as
we will see, are slightly more important than the
square ones.)

(You'll notice that the finished model is somewhat flexible,
in the sense that it doesn't hold its shape rigidly.
Adding 12 more same-sized pencils as vertices,
all meeting at a central point
will solve that problem and make it supremely stable.)

adding 12 vertices will
make it stable and rigid
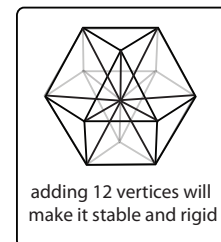
The vector equilibrium is the heart of Synergetics

Bucky's two *Synergetics* books don't include much about the Dymaxion Car, the Dymaxion House, Bucky's World Map, or all the geodesic domes that were built around the globe. The heart of *Synergetics* is geometry. And at the heart of Bucky's geometric discoveries is the **vector equilibrium**, or the cuboctahedron.

It's hard to see the cuboctahedron's shape by looking at a drawing of a cube and an octahedron intersecting.

A cuboctahedron
is the intersection
of a cube and
an octahedron

Bucky's Vector Equilibrium
or the cuboctahedron

Viewing it straight- on
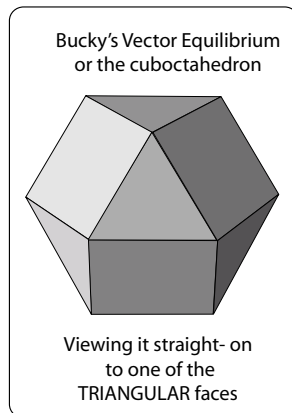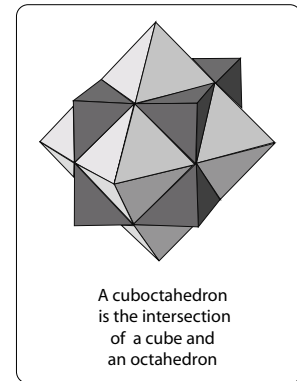to one of the
TRIANGULAR faces

The clearest depiction of a cuboctahedron is viewing it "straight-on" to one of its triangular faces.

Viewing the cuboctahedron "straight-on" to one of its square faces doesn't nearly visually describe it as well. The sense of depth is lost, and there's no clue that the triangles are equilateral triangles.

Despite that, this is a very useful viewpoint (because the 4 triangular faces are so symmetrically arrayed) as you will see in upcoming illustrations.

Bucky's Vector Equilibrium
or the cuboctahedron

Viewing it straight- on
to one of the
SQUARE faces
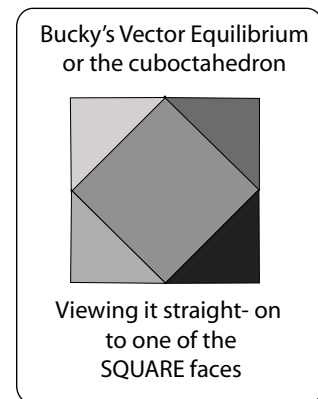
Both views can obtained by slicing the corners off a cube.
It's how you start out viewing the cube that makes the difference.

A cuboctahedron viewed
"straight-on" to a **triangular** face"

Looking "upwards" at a cube

Slice here

The front corner is cut off

More slice marks

All of the corners are cut off, resulting in a **cuboctahedron**

A cuboctahedron viewed
"straight-on" to a **square** face

Almost a straight on view of a cube

Straight-on view of a cube

Slice here

One corner is cut off

All of the corners are cut off resulting in a **cuboctahedron**

To help relate these two views,
I've added a third, "in between" view.

Three views of the same cuboctahedron

"straight-on"
to a triangular face

an
"in between"
view

"straight-on"
to a square face
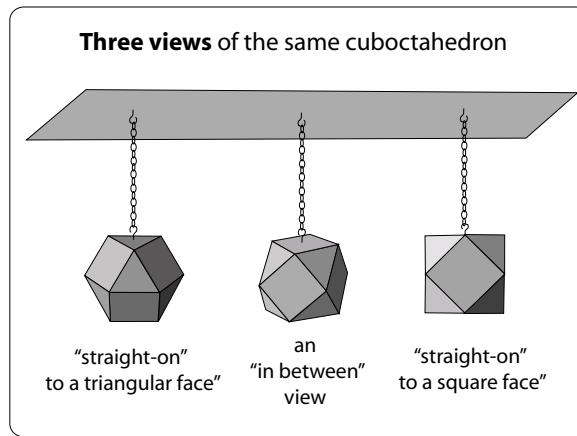
The vector equilibrium is an energy event

Because I'm dealing in the history of gemetry in this book, I use the terms cuboctahedron and vector equilibrium somewhat interchangeably. But Bucky preferred his own coined term, which better depicts it as an "energy event."

He notes that "crystallographers and geometers" called it a "cuboctahedron," but that was in the "non-experimentally informed and non-energy-concerned past." (Fuller, Synergetics 1, 430.04, p.152)

To Bucky, the vector equilibrium wasn't just an interesting geometric shape, it was Nature's energy system. Energy radiation outward or "explosion" was always balanced by energy gravitation inwards or "impulsion." As Bucky dramatically declares,

"Equilibrium between positive and negative is zero.
The vector equilibrium is the true zero reference
in energetic mathematics.
Zero pulsation in the vector equilibrium
is the nearest approach
we will ever know of eternity and god…"
(Fuller, Synergetics I, 440.01, p. 155)

That's a pretty powerful statement about something that he seems to be calling a "zero." Furthermore, it's *invisible*:

"The vector equilibrium is the
anywhere, anywhen, eternally regenerative
event inceptioning and evolutionary accommodation
and will never be seen by man in any physical experience.
Yet it is the frame of evolvement."
(Fuller, Synergetics 1, 440.04, p. 156)

When a vector equilibrium is be modeled with lollipop sticks or ping pong balls, it feels real. It doesn't seem like an "invisible energy equilibrium" that Bucky describes. Let's explore what he means by "zero" and "invisible."

82

The vector equilibrium derives from a the 4-way "pumping" of a tetrahedron

Bucky saw the vector equilibrium as an expression of energy emanating from a tetrahedron, Nature's simplest 3-D shape.

In a chapter entitled "Inside-Outing of a Tetrahedron," Bucky writes,

> "The tetrahedron is the only polyhedron,
> the only structural system
> that can be turned inside out and vice versa
> by one energy event."

Bucky demonstrates 4 tnereffid (different) ways to get a feeling for this "inside-outing" of a tetrahedon by demonstrating four different "pumping models."

Pumping Model 1
"the 3 petaled flower bud"

The first method involves seeing the tetrahedron split apart at the seams and "open up like a 3 petaled flower bud." In this sequence, the "upward pointing" white tetrahedron morphs into a "downward pointing" black tetrahedron.

Pumping Model 2
plunging elastic "edges" through a steel triangle

Bucky welded together three steel rods into a triangle (like the percussion instrument, only fully closed). He attached rubber bands to each of the three corners, then interconnected them in the center of the triangle.

Holding that center conjunction point in his fingers, he would plunge his hand deep into the triangle forming a tetrahedron (made from 3 rubber band edges and 3 steel edges).

Then he would quickly pull his hand back out of the triangle, making a tetrahedron pointed in the opposite direction.

He would plunge his hand back and forth, continuously inside-outing the tetrahedron. This pumping action formed what he called "positive and negative tetrahedra" and demonstrated "the essential twoness of a system."

(Fuller, Synergetics 1, 624.01- 02, p.341)
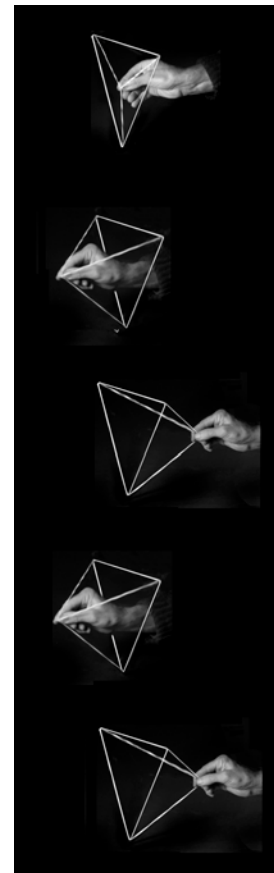
Pumping Model 3
an "involuting" and "evoluting" sphere

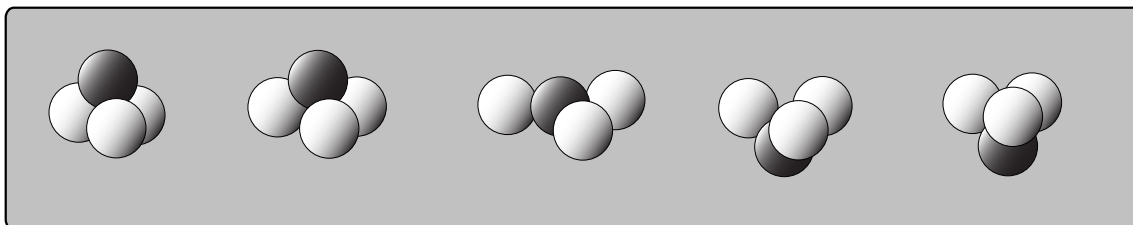This pumping model starts with a tetrahedral arrangement of 4 spheres. Here, a black sphere rest in a nest of 3 white ones.


As the 3 white spheres start to spread apart, the black sphere slowly lowers. When the 3 white spheres have drifted apart far enough, the black sphere is at their level . As the black sphere lowers more, the 3 white spheres start to close up.

When they finally touch again, the black sphere is in their "underneath nest." An "upwards-pointing" tetrahedron has morphed into a "downward-pointing" one.

As Bucky puts it, a tetrahedron made of spheres "turns itself inside out" when it is allowed to "swallow involutingly the fourth sphere through the other three's central passage and to extrude it evolutingly outward again on the other side."

Pumping Model 4
shrinking to a point, then enlarging from a point
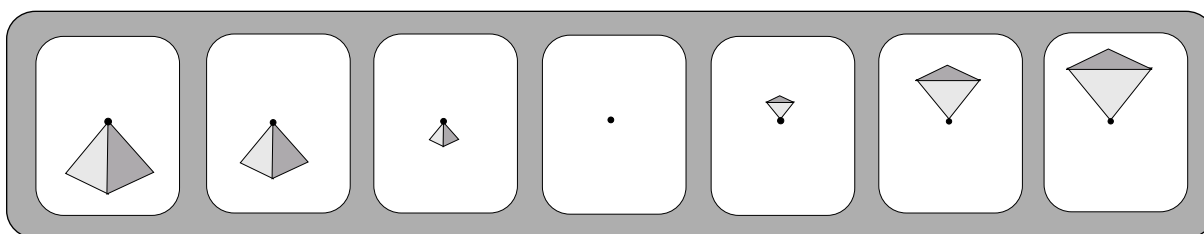
The first frame shows a tetrahedron suspended in space, with its "upper vertex" highlighted with a dot. As the incredible shrinking tetrahedron gets smaller, imagine that the highlighted point stays in the same position in space. Soon the tetrahedron shrinks down to the size of a point.

But a point has no size at all, it's infinitely smaller than infinitely small. Even though it is "one" point, size-wise it's like a "zero." The tetrahedron has essentially disappeared. It has become invisible.

But the story doesn't end there. Suddenly, the tetrahedron pops out the opposite side of the point and starts to enlarge. It gets bigger and bigger until it grows back to it's original size.

To me, this is Bucky's best pumping model, as it best depicts what is happening with the radiating vectors.

When the original tetrahedron shrinks, its "front" vertex travels in a straight line through the centerpoint, then emerges to become the "rear" vertex of the newly enlarged tetrahedra.

The left vertex travels through the centerpoint and becomes the right vertex. Similarly, that right vertex becomes the left.

Energy has flowed in naturally, in straight lines. And all the lines crisscross in that centerpoint.

front edge becomes the back edge

left edge becomes the right edge

right edge becomes the left edge
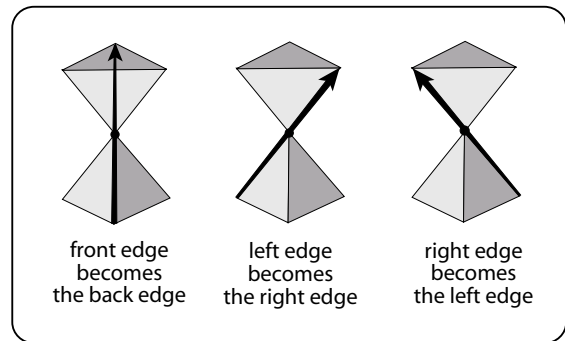
Bucky actually uses the term "bowtie" for a related tramformation (folding 4 great circles ito a vector equilibrim), but this shape is so central to understanding the inner workings of a vector equilibrium, I refer to it as the "**Bucky Bowtie.**" These are not simply two tetrahedra touching, they **must** be oriented so that the vertical edges form straight lines, as this model shows:

Bucky frequently wore a bowtie
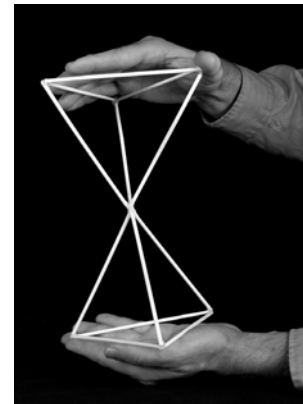
Let's take Bucky's next quote in two parts:
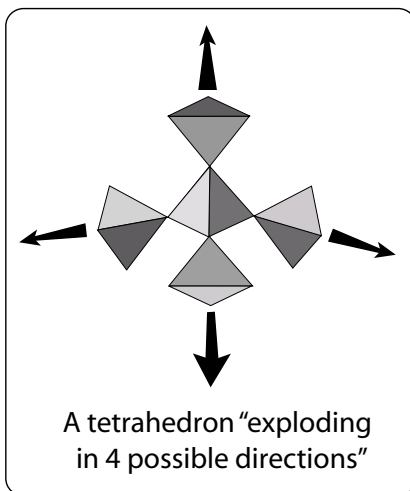
"The vector equilibrium
is a tetrahedron
exploding itself,
turning itself inside out
in four possible directions...

A tetrahedron "exploding in 4 possible directions"

...So we get eight:
inside and outside
in four directions.
The vector equilibrium
is all eight of the potentials."

(Fuller, Synergetics 1, 441.02, p. 157)

"inside and outside
in 4 directions"
or 4 Bucky Bowties

When these 4 Bucky Bowties are pulled apart,
then rejoined so they share a common point,
we have the vector equilibrium,
eight tetrahedra sharing a common point.

(Here's where that
"straight-on" to a square face view
comes in handy.)

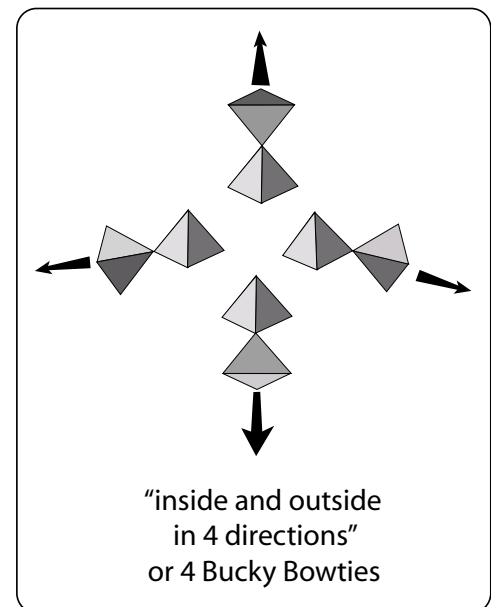Viewing it straight-on
to one of the
SQUARE faces

=

Viewing it straight-on
to one of the
TRIANGULAR faces

Bucky Bowties reunite
forming
vector equilibrium

How the
4 Bucky Bowties
fit in the
vector equilibrium.

Here's another depiction,
showing how the 4 Bucky Bowties
fit in the vector equilibrium:
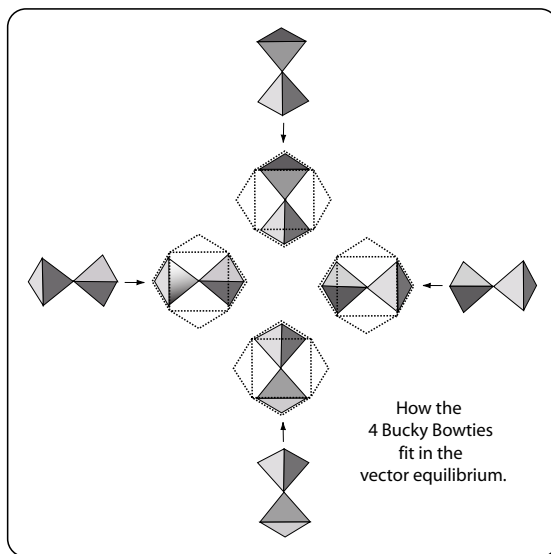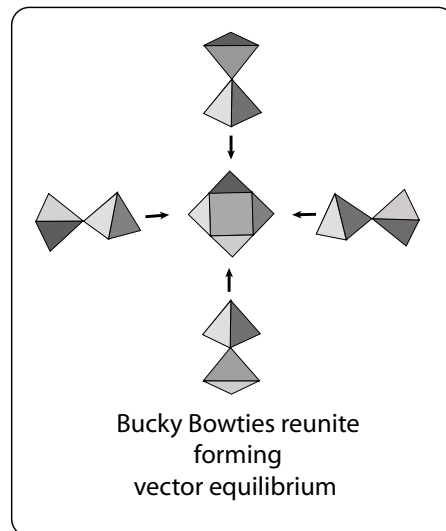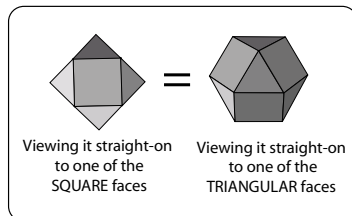
Looking "straight-on" to a square face,
the front view and the rear view look the same.
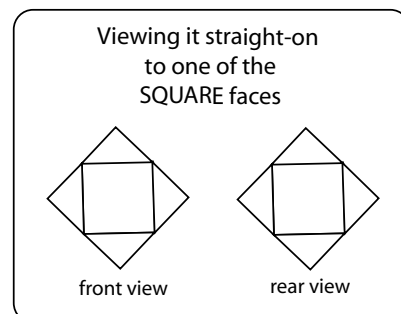
But looking "straight-on" to a triangular face,
everything gets inverted.

The upwards-pointing triangle in the front view,
points downward in the rear view,
as they are both part of the same Bucky Bowtie.

Viewing it straight-on
to one of the
SQUARE faces

front view   rear view

Viewing it straight-on
to one of the
TRIANGULAR faces

front view   rear view
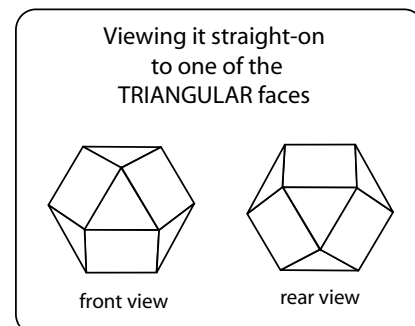
The Four Pumping Models demonstrate
(each in its own way)
what Bucky means when he says:

...each of its four vertices can be plunge-passaged
through its innards...and produces
eight common nuclear-vertex tetrahedra
of the vector equilibrium."

(Fuller, *Synergetics 1*, 481.00-482.00)
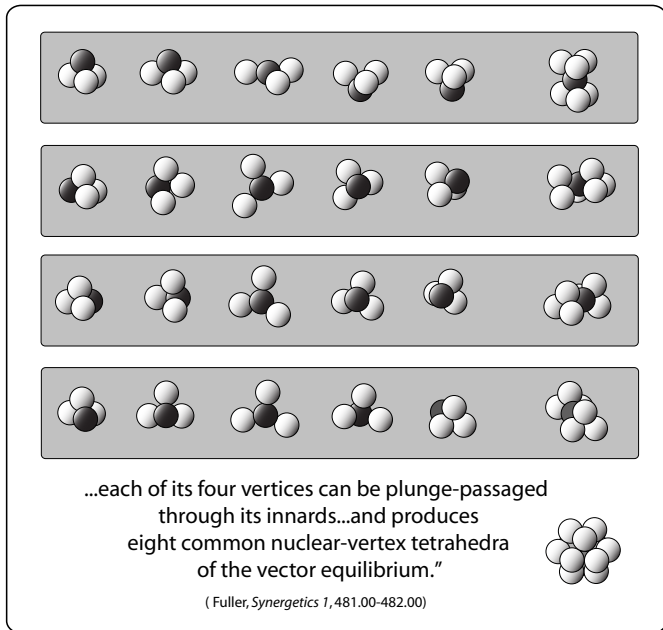
For example, here is the Pumping Model
of the "involuting" and "evoluting" spheres,
which create 8 "tetrahedra of spheres"
which comprise a "vector equilibrium of spheres."

4 Bucky Bowties make a
vector equilibrium

To Bucky, the vector equilibrium was
4 pairs of tetrahedra meeting at a point.

You can see that vector equilibrium is more about
the 8 tetrahedra than the 6 half-octahedra.

To portray this, in this model I've highlighted
the tetrahedra in white
to contrast them with
the half-octahedra shown in black.

"This locus of vanishment is the nearest
to what we mean by a point.
The point is the macro-micro switchabout
between convergence and divergence."

(Fuller,Synergetics 1, 1012.33)

This stylized depiction helps show
the 8 "shrinking and expanding" tetrahedra
as 4 pairs of opposites.

To Bucky, the vector equilibrium was not a shape,
but a *dynamic system*,
always in motion,
a pathway of energy.

These tetrahedra are on the move,
continuously pulsing back and forth,
back and forth, back and forth
through that central point.

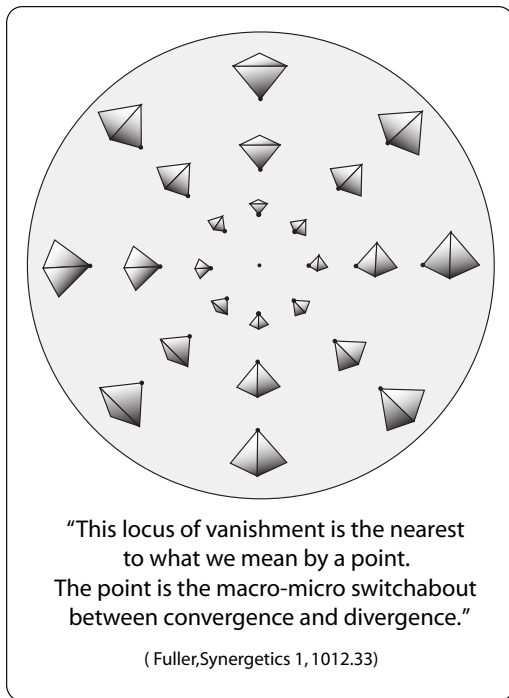That sizeless central point is the key to it all.
Bucky poetically refers to it as the
"**locus of vanishment**"
or the
"**inside-out black hole tetra void**."

(Fuller, *Synergetics 2*, 1033.654 p.401)

This sequence shows
all 8 tetrahedra shrinking
into that point, at the same rate.
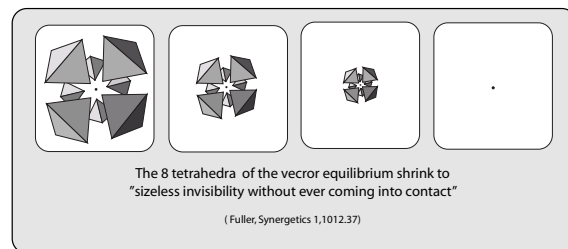
Then they are pulse back out the opposite sides.

Repeat this process several times
and you get a feel for the energy path.

It's kind of like 4 crisscrossing accordians
all being played at the same time,
and when fully squeezed inwards,
they disappear into thin air.

The 8 tetrahedra of the vecror equilibrium shrink to
"sizeless invisibility without ever coming into contact"

(Fuller, Synergetics 1,1012.37)

They also expand outwards ftom that sizeless point

"What we speak of as a point
is always eight tetrahedra
converged to no size at all"

(Fuller, *Synergetics 1*, 1012.33)
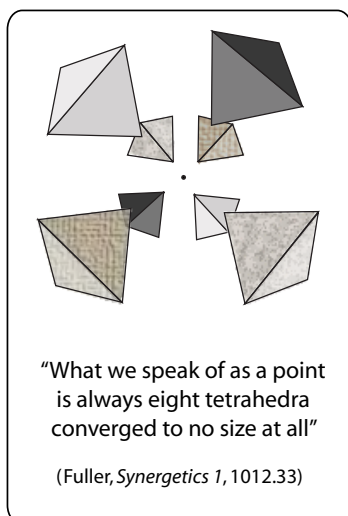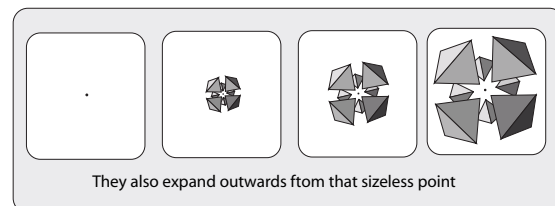
Here, I've differentiated the 4 pairs of tetrahedra
with various tones and textures.
The larger tetrahedra in the foreground
each correspond to their mates in the background.

Bucky felt so strongly about this energy pattern,
he even uses it to define a **point**!

He also saw this point as a **number**.
Can you guess which number?

This is the beginning of Bucky's sub-chapter titled
"Nucleus as Nine = None = Nothing" :

"Nucleus as Nine,
i.e., non (Latin);
i.e., none (English);
i.e., nein (German);
i.e., neuf (French);
i.e., nothing;
i.e., interval integrity…"

(Fuller, *Synergetics 1*, 1012.01, p. 647)

He's pretty clear about it.
To Bucky, that "locus of vanishment" is the **number 9**.

He refers to 9 as "zero-nine" or "null nine,"
suggesting it acts like emptiness, or just plain zero.

He calls the vector equilibrium the
"modular domain of the "nine-zero-punctuated octave system."
All of a sudden, Bucky is associating Geometry and Number.
To understand what he's getting at, let's see how he perceived the realm of numbers.

Numerology

The final chapter of *Synergetics* is entitled called Numerology.
However, Bucky's "Numerology" wasn't about fortune telling,
it was, quite simply, the study (-ology) of Number.
He emphasizes:

"All numbers have their own integrity."

(Fuller, *Synergetics 1*, 1220.10, p. 756)

Bucky demonstrates how this "Octave, null-nine" rhythm can be found in the realm of numbers, as well as in geometry.

He reviews a mathematical technique he used when he worked for Armour and Company in Manhattan in 1920 when he was 25. "The auditors showed us how to check our multiplication by casting out nines."

In this era before hand calculators and even before electric adding machines, they used pencils and did "long multiplication." "Casting out nines" was a form of checking their calculations. (Perhaps you learned it in math class).

It involves boiling a number down to its "digital sum" (i.e. the sum of the digits). Bucky called it or finding the "**indig**" of a number (the integration of its digits).

Here are the rules:

1 Any time the digit 9 appears, cast it out (i.e. cross it out).
2 Any combination of digits that add up to 9 can also be crossed out.
3 Add up the digits that remain.
4 If the result is a two-digit number, add those two digits together, so it boils down to a single-digit number.

Examples of indigging:

24 indigs to 6

913 indigs to 4

90909 indigs to 0

6372815 indigs to 5

4678 indigs to 25, which further indigs to 7

Apply these rules to the 2 numbers involved in the multiplication problem to be checked, then multiply the two single digit results together.

Finally, boil that result down to a single-digit number.
If the resulting number is the same as the number
derived from indigging the product of the multiplication problem,
chances are your original long multiplication was done correctly.
Here's an example:

Checking multiplication by "casting out nines."

7 2 9 4 2   4 + 2 = 6
X   5 3 9 6   = 5   }   6 X 5 = 30
                                            3 + 0 = 3
4 3 7 6 5 2   These two results
6 5 6 4 7 8   should be the same
2 1 8 8 2 6   if the original
3 6 4 7 1 0   long multiplication
                                                      was done correctly.
3 9 3 5 9 5 0 3 2   5 + 5 + 2 = 12   1 + 2 = 3

Here's an example of using "casting out 9's" to check addition

Checking addition by "casting out nines."

4 5 2 7 3 9 2   3 + 2 = 5   These two results
5 8 1 3 9 6 1   5 + 1 = 6   } 5 + 6 = 11   should be the same
                                    1 + 1 = 2   if the original
1 0 3 4 1 3 5 3   1 + 0 + 1 = 2   addition was
                                                 done correctly.

"Casting out nines" can alo be used to check division and subtraction problems.

It must be noted that "casting out nines" doesn't catch every error. Here is an obviously incorrect multi-plication that appears to be right because the product just happened to indig to the right number.

13   1 + 3 = 4
X   2   = 2   } 4 x 2 = 8
44   4 + 4 = 8

Briefly, the reason casting out of nines works is becuases 9 happens to be 1 less than our Base number 10. (If we used a Base 9 numbering system, we would cast out eights)

Examining this "casting out nines" procedure , Bucky succinctly concludes:

"From this I saw that nine is zero."

Bucky applied this "indigging" technique to the normal flow of numbers and found another expression of that "octave, null nine" rhythm.

Indigging
the normal flow of numbers
reveals an "octave, null nine" rhythm.

1 = **1**	10 = **1**	19 = **1**	28 = **1**	37 = **1**
2 = **2**	11 = **2**	20 = **2**	29 = **2**	38 = **2**
3 = **3**	12 = **3**	21 = **3**	30 = **3**	(...)
4 = **4**	13 = **4**	22 = **4**	31 = **4**	
5 = **5**	14 = **5**	23 = **5**	32 = **5**	
6 = **6**	15 = **6**	24 = **6**	33 = **6**	
7 = **7**	16 = **7**	25 = **7**	34 = **7**	
8 = **8**	17 = **8**	26 = **8**	35 = **8**	
9 = **0**	18 = **0**	27 = **0**	36 = **0**	

Next, he indigged of the **squares** of the normal flow of numbers, and found that "octave, null nine" pattern once again.
Note especially that the first 4 results (1, 4 0, 7), are the "opposite" of the next 4 results (7, 0, 4, 1).

Indigging the SQUARES
of the normal flow of numbers
reveals an "octave, null nine" rhythm.

1 = **1**	100 = **1**	361 = **1**	784 = **1**	1369 = **1**
4 = **4**	121 = **4**	400 = **4**	841 = **4**	1444 = **4**
9 = **0**	144 = **0**	441 = **0**	900 = **0**	(...)
16 = **7**	169 = **7**	484 = **7**	961 = **7**	
25 = **7**	196 = **7**	529 = **7**	1084 = **7**	
36 = **0**	225 = **0**	576 = **0**	1089 = **4**	
49 = **4**	256 = **4**	625 = **4**	1156 = **0**	
64 = **1**	289 = **1**	676 = **1**	1225 = **1**	
81 = **0**	324 = **0**	729 = **0**	1296 = **0**	

Then he applied it to the **cubes** of the normal flow of numbers, and still found that "octave, null nine" pattern.

Indigging the CUBES
of the normal flow of numbers
reveals an "octave, null nine" rhythm.

1 = **1**	1000 = **1**	6859 = **1**	21952 = **1**	50653 = **1**
8 = **8**	1331 = **8**	8000 = **8**	24389 = **8**	54872 = **8**
27 = **0**	1728 = **0**	9621 = **0**	27000 = **0**	(...)
64 = **1**	2197 = **1**	10648 = **1**	29791 = **1**	
125 = **8**	2744 = **8**	12167 = **8**	32768 = **8**	
216 = **0**	3375 = **0**	13824 = **0**	35937 = **0**	
343 = **1**	4096 = **1**	15625 = **1**	39304 = **1**	
512 = **8**	4913 = **8**	17576 = **8**	42875 = **8**	
729 = **0**	5832 = **0**	19683 = **0**	46656 = **0**	

(after Fuller, *Synergetics 1*, Indig Tables, Fig. 1223.12, p. 767)

Bucky found that the octave had its own internal rhythm.
In the example of the indigging resuts of the squares of the normal flow of numbers,
you can see what he calls the "+4, −4" nature of the octave.

| NULL ZERO | OCTAVE +4 −4 | NULL NINE | OCTAVE +4 −4 | NULL NINE | OCTAVE +4 −4 | NULL NINE |

As Bucky puts it,
"Indig congreunces demonstrate that nine is zero
and that number system is inherently octave..."
with an internal rhythm of " four positive and four negative."
He summarizes it as:

"The inherent +4, −4, 0, +4, −4, 0 ⟶ of number"

(Fuller, *Synergetics 1*, 1222.11-12, p. 764)

This rhythm he found in number corresponded
with the rhythm he saw in geometry.
The vector equilibrium is made
from 4 pairs of tetrahedra
all intersecting at
the "null nine"
centerpoint.

4 pairs of tetrahedra
and the
centerpoint is "null nine"

The "fourness" of a tetrahedron

4 vertices   or   4 faces   or   4 spheres

Bucky also corresponds this "fourness" in number
to the tetrahedron,
"the prime structural system of Universe."
It might be seen though as the
4 corner points (vertices) of the tetrahedron,
or as its 4 sides,
or even as a tetrahedral cluster of 4 spheres.

He declares,
"There are apparently
no cosmically absolute numbers
other than 1, 2, 3, and 4."

In a sense, 5, 6, 7, 8, are "a reflection" of the cosmically absolute 1, 2, 3, and 4.
There is a pause at 9.
Then the pattern keeps repeating itself into the number realm.

Amazingly, Dee seems to have seen the same thing 400 years earlier.
His graphic depiction of the octave in "Thus the World Was Created" chart
is made from 2 groups of 4 digit each (1, 2, 3, 4) and (5, 6, 7, 8).
And the "null nine" is the "Horizon of Eternity,"
sits just above this octave.

The" large, dotted-line X" seems to be relating
the two quaternaries to each other,
incorporating the idea of oppositeness.

This concludes the brief tour of Bucky's 10 Considerations.
It's been a whirlwind journey from vectors, to the tetrahedron, to the octet truss,
to the closest-packing-of-spheres,
to the "+4, -4, octave; null 9,"
seen in the vector equilibrium
and in the number realm.
But there's one more important aspect of these Considerations that I've been saving for last.

Multiple layers of closely packed spheres

A vector equilibrium is a description of energy radiating outward from a point
(as well as gravitating inward towards that point). Scale doesn't matter.
Model-wise it can be made from toothpicks, lollipop sticks, wooden dowels,
plumbing pipes, or even telephone poles.
Each of its 12 radiating vectors will always be the same length as each of its 24 edge vectors.

But as "Unity is plural and at minimum 2," each vector,
be it toothpick or telephone pole, can be seen as two tangent spheres.
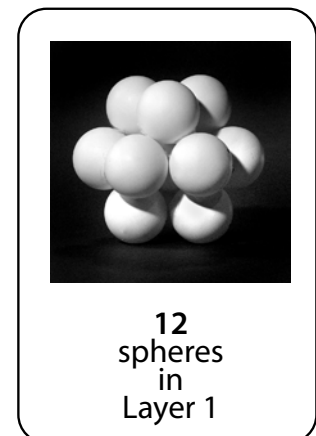
Layer 1

1
central
sphere
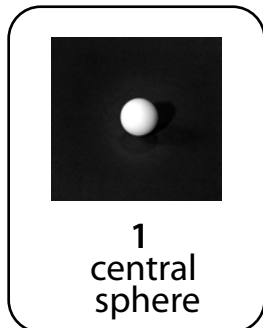
So, the arrangement of 12 raditing vectors,
is the same thing as 12 spheres
symmetrically arrayed around 1 central sphere.

These twelve spheres make up "Layer 1."
How many spheres do you think it takes
to pack perfectly around this cluster in Layer 2?

And what will be the resulting shape?
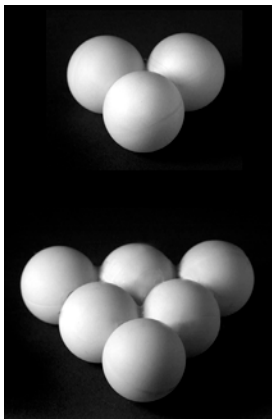

12
spheres
in
Layer 1

93

Layer 2

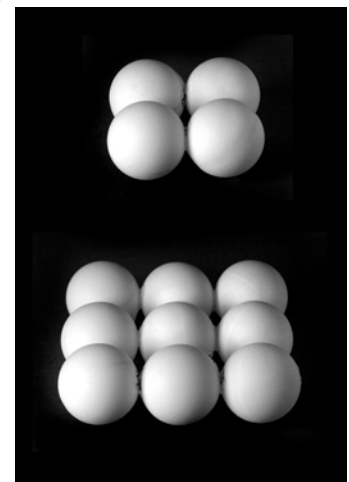Let's first consider
the triangular faces of Layer 1,
each of which has three spheres.
Layer 2 must provide
three "nests" for them,
so the triangular faces of Layer 2
must each have six spheres.

A triangular arrangement
of 6 spheres
makes 3 "nests."

The square faces of Layer 1
each have 4 spheres.
To provide 4 nests,
the square faces of Layer 2
must have 9 spheres.

A square arrangement
of 9 spheres
makes 4 "nests"

Thus, Layer 2 seems to need 84 spheres:

8 triangular faces X 6 spheres each = 48
6 square faces X 9 spheres each = 36
84 in total

But just as the triangular and square
faces of Layer 1 "share" spheres,
a similar "sharing" occurs in Layer 2.

In fact, ALL the spheres on Layer 2 are shared
(except for the single sphere in the center of the square faces
with 9 spheres.)

8 triangular faces X 6 spheres each = 48
6 square faces X 1 sphere each = 6
54 in total

So this means that
only 54 spheres seem to be required.

However, the spheres at the 12 vertices
are each shared by 2 triangular faces.
Thus we can reduce the number 54 by 12,
resulting in **42** spheres.

And as you can see,
the cuboctahedral shape of Layer 1
is maintained in Layer 2.
It's simply larger.
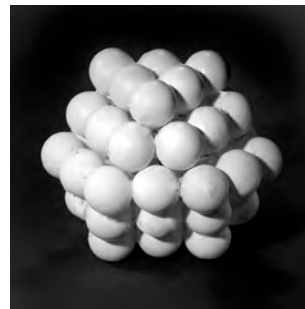
42
spheres
in
Layer 2

We now have a symmetrical conglomeration of 55 spheres.
(1 center sphere +12 spheres in Layer 1 + 42 spheres in Layer 2)
This is a noteworthy arrangement because, for the first time,
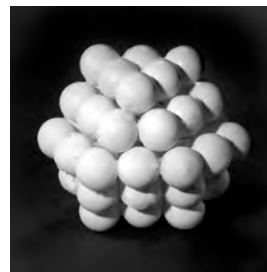the center sphere has become a real nucleus.

With only the first layer, you could peek
in between the spheres and see the nucleus.

But once the second layer has been applied,
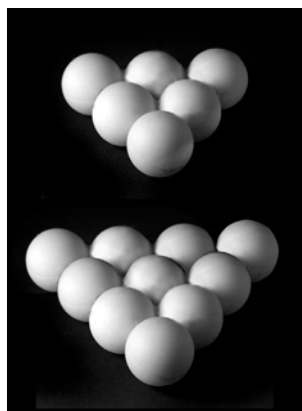those gaps get concealed,
and the center sphere is impossible to see.
This raises its status, and it is now a true "nucleus."

A cluster of 55 total spheres
makes the central sphere
a "true nucleus."

Layer 3

For the triangular faces,
to provide enough nests
for the 6 spheres of Layer 2,
Layer 3 needs to have 10 spheres.

For the square faces,
to provide enough nests
for 9 spheres of of Layer 2,
Layer 3 needs to have 16 spheres.

This first, rough tally amounts to:

A triangular arrangement
of 10 spheres
makes 6 "nests."

8 triangular faces X 10 spheres each = 80
6 square faces X 16 spheres each = 96
176 in total

A square arrangement
of 16 spheres
makes 9 "nests"

But, the triangular and square faces "share" all their spheres,
except for 1 sphere in the center of the triangular faces
and 4 spheres in the middle of the square faces

The new tally is:

8 triangular faces X 10 spheres each = 80
6 square faces X 4 spheres each = 24
104 in total

And again, the triangular faces
share the 12 spheres at the vertices,
so 104 minus 12 makes **92** spheres.
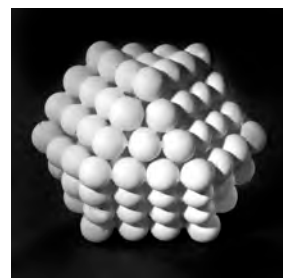
92
spheres
in
Layer 3

Before going on to Layer 4, let's take
a closer look at the triangular faces of Layer 3.
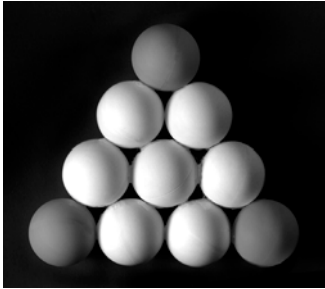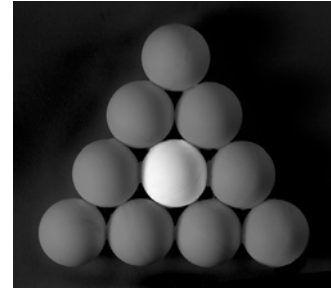

With 10 spheres, it's in the shape of the
Pythagorean tetractys or the "bowling pin arrangement."

Nine of the 10 spheres are all on the edge of the triangle
and 1 is cradled, totally surrounded by the rest of them.

It's the center point of this
two-dimensional-triangle of spheres.

Temporarily remove the three corner spheres
and you'll see it as the center
of a hexagonal, 6-around-1 arrangement.

I am emphasizing this particular center sphere, of this particular layer,
because it is a seed, a "potential" nucleus.

After we have added 2 more layers,
it will have become the center of its own 55-sphere conglomeration.

It's the first opportunity for such a "nucleus"
because Layer 2 and the original center nucleus provide
part of the two-layer's worth of surrounding spheres necessary.

There are 8 such potential nuclei in this "Layer 3 of 92 spheres,"
one in the center of each of the 8 triangular faces of Layer 3.

(All this will become clearer as we add the next two layers)

Layer 4
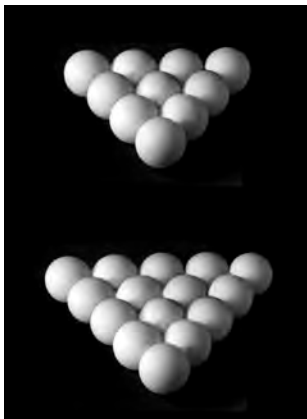
A triangular arrangement
of 15 spheres
makes 10 "nests."

In the triangular faces of Layer 4,
15 spheres make 10 nests
accomodating the 10 spheres Layer 3.
These 15 spheres is the arrangement
pool players make before each game
with the help of a wooden rack
in the shape of equilateral triangle.
Rack 'em up!

In the square faces,
25 spheres make 16 nests,
accomodating the 16 spheres of Layer 3.
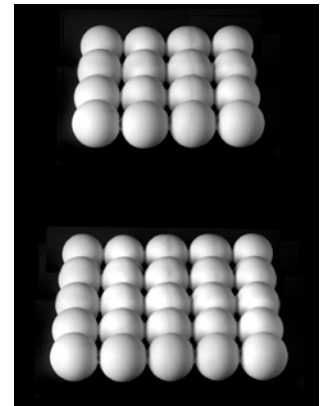

A square arrangement
of 25 spheres
makes 16 "nests"

The accounting,very briefly, is :
120 shared spheres +54 spheres in the central areas of the square faces = 174 spheres.
Minus the 12 vertices, makes **162** spheres total for Layer 4.

Do you notice a pattern in the number of spheres per layer?

You've probably noticed that the number of spheres
in the square faces is the series of square numbers
$(4, 9, 16, 25, 36, 49,…)$
and that the square faces accumulate
like an enlarging Pyramid of Giza
(or a half-octahedron).
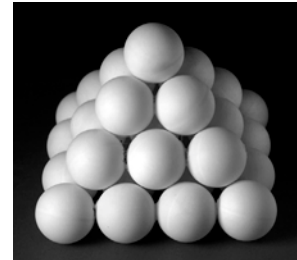
The square faces
accumulate
in shape of an enlarging
Pyramid of Giza
(a half-octahedron)
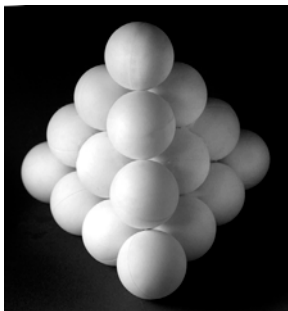
The triangular faces
grow as an
enlarging tetrahedron.

The number of spheres
in the triangular faces
$(3, 6, 10, 15, 21, 28, …)$
are the triangular numbers.
They grow like an enlarging tetrahedron.

You may have also noticed
something peculiar about
the number of spheres per layer:

Layer 1	12
Layer 2	42
Layer 3	92
Layer 4	162

They all end in the number 2!
Let's take away those 2's for a moment.

What remains is $1, 4, 9, 16$.
Hey, thats the sequence of square numbers!
More significantly, they are the squares of the Layer numbers:
Layer 1, Layer 2, Layer 3, Layer 4.

As we had omitted the 2's, what we really have is a sequence of:
10 x (the Layer number "squared"), plus 2

	Ten times (the Layer number "squared")...	...Then add 2.
Layer 1	$10 \times 1 = 10$	$10 + 2 = 12$
Layer 2	$10 \times 4 = 40$	$40 + 2 = 42$
Layer 3	$10 \times 9 = 90$	$90 + 2 = 92$
Layer 4	$10 \times 16 = 160$	$160 + 2 = 162$

This formula for the total number of spheres
in successive layers of closest packing-of-spheres
is credited to the Swiss mathmetician
Leonhard Euler (1707-1783):

But regardless of how many layers are added,
the shape will **always** be a cuboctahedron.

Euler's Formula
for the number of
spheres-per-layer
in the
closest-packing-of-spheres

$$(10\,L^2) + 2$$

L= Layer number

Leonhard Euler

Layer 5

Now, (drum roll please)
how many spheres will be in the
fifth layer of closest packing of spheres?

	Ten times (the Layer number "squared")...	...Then add 2.
Layer 5	10 x25 = 10	10 +2 =252

It's Dee's Magistral Number! 252

That number that seemed so obscure in the *Monas Hieroglyphica*
appears as an important number in this natural growth pattern of close-packing spheres.

Here's why this fifth layer of 252 spheres is such an important layer:

Remember those 8 potential nuclei that were cradled
in the 10-sphere bowling pin arrangement in Layer 3?

Once Layer 5 has been added, they finally become "true nuclei."
They are now totally enveloped by two layers of spheres.
They are nuclei in their own very own 55-sphere clusters.

The 8 spheres are buried so deeply,
its hard to depict them,
but this graphic indicates their approximate positions
with respect to the original nucleus.

98
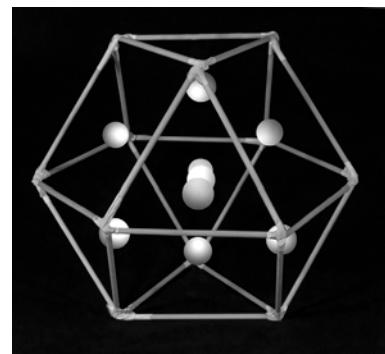
These 8 "newborn nuclei," as Bucky calls them, are symmetrically arrayed around the original central nucleus in another geometric expression of "**octave, null nine**."

Each of them is nestled in one of the 8 "tetrahedral outgrowths"
and they are thus an expression of the 4 pairs of tip-to-tip tetrahedra
that make the cuboctahedron.
Each newborn has a "twin" in the expanding tetrahedron on the opposite side.

In this "blown apart" view,
you can see how these two
(of the "8 new nuclei")
are "a pair."

5 layers of "opposing tetrahedra" in the cuboctahedron

(2 of the "8 new nuclei" are marked with small stars)
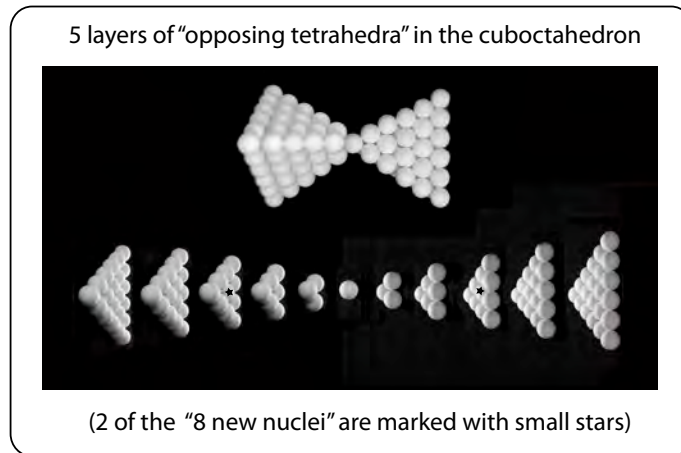
So besides simply expressing "**octave, null nine**,"
the nine nuclei are, more specifically,

an expression of the "**+4, −4, octave; null nine**" rhythm
that Bucky also saw in the realm of numbers.

Layers 6, 7, 8, 9, 10, 11, ...

As per Eulers's formula,
Layer 6 has 362 spheres,
Layer 7 has 492,
Layer 8 has 812,
Layer 9 has 1002, etc.,
but that gets to be a lot of ping pong balls.
In the midst of this swelling cuboctahedron,
more new nuclei aer created,
not just in the centers of triangular faces,
but in the centers of square faces as well.

Dee discovered in the 1500's what Bucky discovered in the1900's

See, Bucky's *Synergetics* isn't so hard to understand after all.
After this brief romp through the mind of a great thinker from the mid 1900's,
let's flip back in time 4 centuries.

In the mid 1500's, John Dee had figured all this (and more), all by himself!
Synergetics is at the core of what Dee is trying to express (cryptically)
in his **Monas Hieroglyphica**!

John Dee was an Atomist

252

Dee's Magistral number is 252 and number of spheres in the fifth layer of closest packing of spheres is 252. But is hardly enough evidence that Dee was aware of the cuboctahedral arrangement of closest packing of spheres. Let's look for more clues.
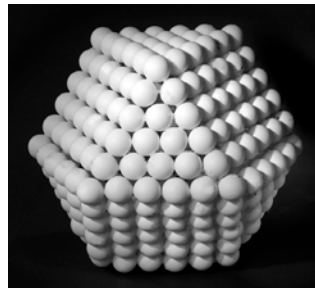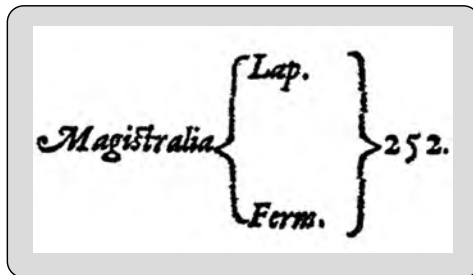
252 spheres in the fifth layer of closest packing of spheres

The Numbers "12, 13, 24, 25" in the Monas Hieroglyphica
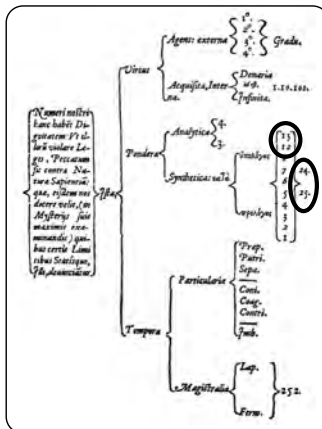
As mentioned earlier, Dee emphasizes the pairs of number "12, 13" and "24, 25" in both of his summary charts.

An astute geometer would recognize that these pairs of numbers relate to numbers in the accounting of various parts of the **cuboctahedron**. Exactly **12** spheres close-pack perfectly around 1 central sphere (12, 13).

Connecting the center points of the actual spheres with vectors (lines, dowels, or lollipop sticks) makes 12 lines radiating to 12 outer vertices from one central point (12 +1 = **13**).

To connect these outer vertices requires **24** outer edges.

(Actually, 4 edges meet at each of the 12 vertices, making it seem like there are 48. But, as each line is "shared" with another vertex, the total edges is half that, or 24.)

As we've seen, when this framework of 24 edge lines is projected outwards onto a sphere, it makes a spherical cuboctahedron. The total number of great circles that pass through the vertices, face, center points, and edge midpoints totals **25** great circles.
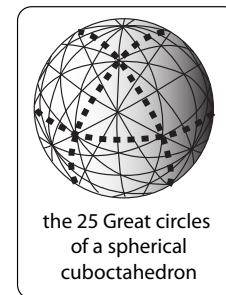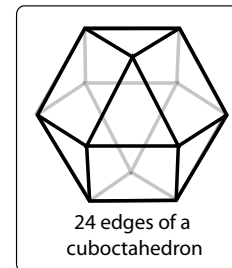

"12 around 1" makes 13 in total

24 edges of a cuboctahedron

the 25 Great circles of a spherical cuboctahedron
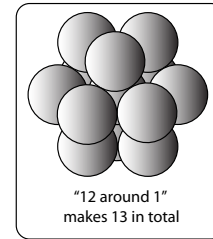
These 25 great circles might seem like an obscure factoid that few people would know about. However, any scholar involved with cartography or navigation would be well-versed in great circle accounting. Dee was an expert in both areas. He advised all the great Elizabethan sailors on navigation and drew one of the first circumpolar maps to help guide them in their quests to find the Northeast and Northwest passages.

To summarize, the two pairs "12, 13 and 24, 25" say "cuboctahedron," but primarily in two different ways. Though the numbers 12, 13 can relate to the 12 vertex points around a center point, they would be more recognizable as 12 spheres around a central sphere, a cuboctahedron made from **spheres**. The numbers 24, 25, on the other hand, relate to edges and great circles, or a cuboctahedral shape made from **line**s.

So, it seems as though Dee was aware of cuboctahedrons made from spheres as well as made from lines, as well as the connection between the two. But these 4 numbers alone are not solid enough proof of this.

For more clues that Dee knew about the cuboctahedron, first we'll investigate the **history** of closest-packing of spheres. Then, we will turn to Renaissance knowledge of the cuboctahedron made from **lines.**

Dee's graphic clue about closest packing of spheres:
(eagle's eggs and dungballs)

In the 24 Theorems of the *Monas*, Dee only relates one parable: Aesop's fable of the Eagle and the Dung Beetle. As you recall, the Jupiter allowed the Eagle to place its eggs in his lap to keep them safe from the vengeful Scarab Beetle. But the clever Beetle managed to deposit some of his spherical dungballs in with the eggs.

Hmm, all these spherical objects nestled together in Jupiter's lap. . . . How do you think they arranged themselves?

They would arrange themselves according to the natural law of **closest packing of spheres**. It doesn't matter if its atop Mt. Olympus in 5,000 B.C.or in Timbuktu in 2002. It's Nature's organic Law of Arrangement.

Granted, it's not known if there were a total of 13 eggs and dung balls, but even if there were fewer, they would be close-packed in tetrahedral (1 in a nest of 3) or half-octahedral (1 in a nest of 4) arrangement, which, if extended, would be the cuboctahedral closest-packing arrangement.

Dees big hint here is when he tells King Maximillian, "I am not trying to play Aesop, but Oedipus." He has related the fable, not for its moral message (Aesop), but because it is a riddle (Oedipus). The solution to the riddle is that the eggs and dung balls close-pack cuboctahedrally. You still might consider my analysis conjectural, so let's look at a bigger picture of Dee's philosophy: **Dee was an Atomist**.

A Brief History of Atomism

Atomism is simply the theory that the universe is made from atoms, which can neither be divided nor destroyed. But Dee didn't invent this concept by any means.

Atomism goes back to the Greek philosophers:

Anaxagoras (ca. 500 BC – 428 BC),
Leucippus (ca. 425 BC),
his student **Democritus** (ca. 375 BC),
and, later, **Epicurus** (ca. 300 BC).

Dee knew of these early philosophers by reading *Lives, Teachings, and Sayings of Famous Philosophers by Digenes Laertis,* who compiled a history of Greek philosophy around 250 AD. Dee owned two copies of Laertius' popular text.   (Roberts and Watson, 502 and 1069).

Diogenes reports that Anaxagoras believed that:

"the primary elements of everything were similarities of parts.
For as we say that gold consists of quantity of grains combined together,
so too is the universe formed of a number of small bodies of similar parts."

He further taught that "Nous" (or Intelligence or Mind or Reason)
started with a revolving motion that is continuous and eternal.

Anaxagoras felt that:

"Heavy bodies, such as the earth, occupied the lower situations.
The light ones, such as fire, occupied the higher places.
And the middle spaces were assigned to water and air."

(Diogenes Laertius, *Anaxagoras*, I 14, 42)

Anaxagoras believed that because nothing really just pops into being, everything must be contained in everything else in the form of infinitely small parts. The Nous, or Intelligence, set all the particles in a whirling motion and the parts all separated, then re-combined in various ways to make all the stuff we see around us. The Universe is a in a Waring blender.

Dee writes about Anaxagoras in the Monas

Dee hints at to Anaxagoras' conception of a whirling motion re-distributing all the small parts in the "Metamorphosis of the Egg" or the "Spiral Diagram" in Theorem 18. The crushed shells, yolk, and white of the egg are being mixed by swirling "Spiral Revolutions." In the text Dee writes, "Later, Anaxagoras made his most excellent Medicine from this Teaching, as seen in his little book *Peri Ton Ekstrophan Physikon* or the *"Nature of Whirling Around Fast."*

C. H. Josten comments, "that no work of Anaxagoras bearing this title has been found." But, Dee is simply referring to Anaxagoras' cosmology in general.

Dee learned of Anaxagoras' ideas from Diogenes Laertius and Plutarch. *In Lives of the Eminent Philosophers*, Diogenes writes:

"Anaxagoran eipien os olos o ouranos ek lithos sygkeoito te sphodra de peridinesei synestanai kai anathena katenextheses thai"

"Anaxagoras says the whole heaven is composed of rocks which band together by whirling very fast in a circle and which would fall down if relaxed."
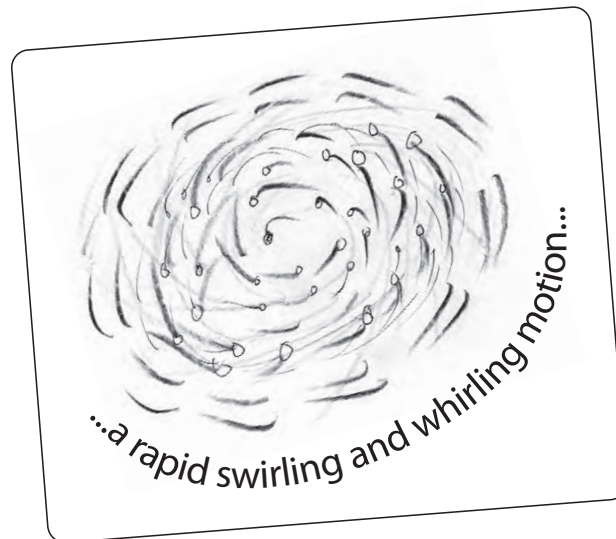
(This is my translation based on Hicks, *Diogenes Laertius I*, Anaxagoras, pp. 142-3)

...a rapid swirling and whirling motion...

This agrees with Plutarch's comments about Anaxagoras in the chapter on the "Life of Lysander" in *Parallel Lives* in a discussion about a large meteorite that fell near Aegospotami, in the year 468 BC (now Gallipolie Peninsula, Turkey, about 150 miles southwest of Istanbul).

**"... a stone of vast size had fallen from heaven at Aegospotami,
and it is shown to this day by the dwellers
in Chersonese, who hold it in reverence.**

**Anaxagoras is said to have predicted that if
the heavenly bodies should be loosened
by some slip or shake, one of them might be torn away,
and might plunge and fall down to earth.**

And he said that none of the stars was in its original position.

**For being of stone, and heavy, their shining light
is caused by friction with the revolving aether,
and they are forced along in fixed orbits
by the whirling impulse which gave them their circular motion,
and this was what prevented them from falling to our earth in the first place..."**

(Loeb, Plutarch's *Parallel Lives*, Chapter 12)

The Great Greek Atomists

Aristotle claims that Leucippus, one of Anaxagoras' contemporaries, was actually the originator of atomism. Diogenes reports that Leucippus' view was that "einai" (which is Being, Existence, or The All) is part "pleres" (**the Full**) and part "kenon" (**the Empty**). When atoms in the void start whirling around and become entangled with each other, they form the "substance of the stars."

Leucippus' most famous pupil, Democritus, elaborated on atomism around 375 BC.
He concludes,

**"In nature, there is nothing but
'atoma' (atoms)
and
'kenon' (void, space, emptyness).**

The word "atom" comes from the Greek word "atmos," meaning "indivisible." The prefix "a-" means "not" and the word "temein" means "to cut." These **"uncuttable"** particles are actually so small the eye can't see them.

Atoms are "solid," "unalterable," and there are an "unlimited number of them."
They swirl around in a *dineuma*, a vortex or a circular rotation like dust in a tornado or like bubbles in the eddy of a stream. (*Dinos* also means a small, round threshing floor where oxen were put to crush corn.)

Democritus felt that the Sun and the Moon and "The All" were comprised of these "smooth and spherical masses." Swirling atoms are "the cause of the creation of all things."

All composite things, like fire, water, air, and earth are *sustemata*, meaning "a bringing together," (from which we derive our word system.) They are all various conglomerations of atoms.

(*Diogenes Laertius*, Hicks, Vol II, p. 453-5)

A generation later, **Epicurus** (341 BC–270 BC) was also an ardent atomist. But felt that atoms had a little more free will. He felt atoms didn't only travel in lines, but could exhibit a *klinamen*, meaning "a swerve, bend or change of direction." This is reflected in his Epicurean philosophy human free will and "maximizing happiness of oneself and others." Breaking tradition, he admitted women and slaves into his school. His ideas on freedom have influence philosophers over the ages, including John Locke and Thomas Jefferson. (Wikipedia, "Epicurus", pp. 1-5)

Another storng exponent of atomism was the Roman poet **Lucretius** (ca. 100 BC–55 BC), one of the first Epicureans to write in Latin. The first three books of his six book epic philosophical poem *De Rerum Natura* or *On the Nature of Things*, portrays being and nothingness as atoms and spaces. (Dee owned two copies of Lucretius' work as well as the famous commentary on it by Jean Baptiste of Bologna.)

While most of the focus of atomisms was on the atoms, Heron of Alexandria (ca. 62 AD) made use of the "vacuum." However, Heron felt the vacuum existed between particles of matter, not invisible atoms. Heron invented the first steam engine – a boiler that used escaping steam to produce rotary motion. His famous book *Pneumatica* describes siphons, a fire engine, a water organ, and arrangements employing the force of steam to perform work." (Dee owned two books of Heron of Alexandria's works.)

(Roberts and Watson, 383 and 385; and *Encyclopedia Brittanica*, Heron of Alexandria)

Atomism makes a "Renaissance"

These early theories of atomism were largely forgotten for centuries. Around 1415, one of the Pope's secretaries, **Poggio Bracciolini** was rummaging through a monastery library when he came upon a manuscript of Lucretius' *On the Nature of Things*. Editions of Lucretius' poem about "matter and void" soon spread across Europe.

Great thinkers like **Nicholas of Cusa, Marcello Ficino, and Giordano Bruno** were all influenced by Lucretius' ideas about atomism. (Giordano Bruno lived in England for a while, but Dee kept his distance from this controversial character)

The 5 Platonic solids in Kepler's model of the universe

As Andrew Pyle writes in *Atomism and its Critics: from Democritus to Newton*, during the late 1500's and early 1600's, "Atomism was becoming very popular and widespread." By the year 1600, the Classical Atomic theory had been thoroughly revived and was the subject of heated controversy. (A. J. Pyle, pp. 224-5)

We next turn to the German astronomer **Johannes Kepler** (1571–1630), who published *Mysterium Cosmographicum* or *The Mystery of the Cosmos* in 1595, defending the ideas of Copernicus. His model of the universe involved the Platonic solids – an octahedron inside an icosahedron, inside a dodecahedron, inside a tetrahedron, all inside a cube.

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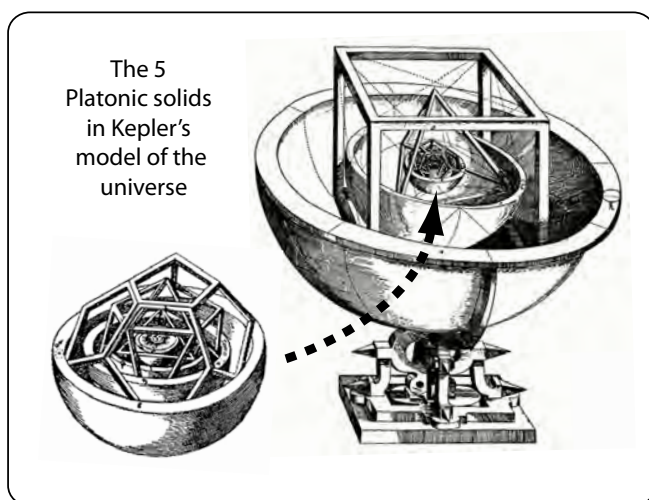
Harriot influences Kepler

In 1601, Johannes Kepler replaced Tycho Brahe as King Rudolph II's imperial mathematician in Prague. Over the next few years, Kepler wrote *Astronomiae Pars Optica* (*The Optical Part of Astronomy*). In it, he discusses the principles of the camera obscura and vision, reflections from parabolic mirrors, atmospherical refraction, and how light behaves according to the inverse square law.

Eager to absorb what others knew about optics, he started to pick the brain of English mathematician and optician **Thomas Harriot** (1560–1621).

During his college years at Oxford, Harriot had developed the reputation of being a scientific prodigy. In 1585, at the age of 25, he was sent on Sir Richard Grenvilles' expedition to Virginia as navigator, surveyor, and scientific advisor.

Upon his return, the wealthy courtier Henry Percy, the ninth Earl of Northumberland (known as the Wizard Earl) became his patron. Harriot was given a pension, a house, and a laboratory at Sion House (near Richmond, about three miles up the Thames from Mortlake). He dedicated his life to scientific pursuits and never was married.

Harriot even hired his own lens grinder, Christopher Tooke, and they built telescopes, or "perspective trunks" as they called them They viewed the moon as well as the moons of Jupiter well before Galileo. Harriot was the first to observe sunspots – again, three years prior to Galileo's observations. (Shirley, Harriot's Biography, pp. 381-3)

Thomas Harriot

Harriot recognized that planets had elliptical orbits, years before Kepler published on this phenomenon. Harriot was an innovator in mathematics. He was the first to use the > "greater than" and the < "less than" signs that we still use today.

Harriot discovered that the ratio of the sines of the angles of incidence and refraction are constant when light passes between two transparent media. (In 1637, Renee Descarte was the first to publish this finding, called Snell's Law, after Willebrord van Ruijen Snell, who discovered it in 1621. Harriot was using it in 1605.) Harriot and Tooke even measured the refractive indices of all the colors of a rainbow created by a glass prism.

Kepler had been working on the refraction of light as well, so in 1605 he wrote Harriot asking about his ideas on what causes the rainbows.

Harriot responded with a short note, but was not very forthcoming about his discoveries. However he did share information about the refractive indices of various liquids like spring water, salt water, Spanish wine, German wine, alcohol, turpentine, and olive oil, as well as solids like rock salt, crystal, and resin. Kepler sent a follow up letter asking Harriot for even more data.

Harriot was slow to respond, finally writing a one page letter.   But he did suggest to Kepler that to understand the secrets of physical optics one must be open to **Atomism**:

"I have now led you to the doors
of nature's house, wherein lie its mysteries.

If you cannot enter because the doors are too narrow,
then abstract and contract yourself into an atom,
and you will enter easily.

And when you later come out again,
tell me what wonders you saw."

(Kargon, p. 24, letter from Harriot to Kepler, in *Kepler Werke, XV*, 368)

In his reply August 2, 1607, Kepler wasn't buying Harriot's ideas of "atmos and vacua." In a later letter, Harriot wrote, "**I confess that my opinion is founded upon the doctrine of a vacuum . . . . But things are such that I cannot as yet freely philosophize.**"

(Kargon, p. 27, Harriot to Kepler, July 13, 1608, *Kepler Werke, XVI*, 172)

In 1605, Harriot had been temporarily imprisoned in connection with his compatriot's activities in Guy Fawkes' attempt to blow up the Houses of Parliament in November of 1605, so he was still a little reserved about expressing his opinions. He tells Kepler, "**we are still in the mud**" here in England, meaning the intellectuals weren't as open to new ideas as they were in Prague.

Kepler's "Six – Cornered Snowflake"
becomes "Kepler's Conjecture"

Eventually, Kepler warmed up to Harriot's ideas that Nature's secrets might be discovered by studying how small spherical atoms pack together, leaving the least amount of void.

In 1611, Kepler wrote a small book on this subject entitled *Strena Seu, De Nive Sexangula* (*A New Year's Gift, on the Six-Cornered Snowflake*) It was a present to his patron, John Matthew Wacker, a court counselor to Rudolph II.

JOHANNES
KEPLER

Kepler had been walking across the famous Charles Bridge
over the River Vlalta in Prague when it started snowing.
He started wondering why is it that:

"snowflakes, in their first falling,
before they are entangled in larger plumes,
always fall with six corners and six rods, tufted like feathers?...

For if it happens by chance, why do they not fall just as well
with five corners or seven?
Why always six ?"

(translated by Colin Hardie, *Kepler*, p. 7)

This got Kepler thinking about honeycombs, hexagons, and then about why six "balls" arrange in the 6–around–1 arrangement. He notes that in many layers, each ball is touched by twelve others. (He notes that the seeds of a pomegranate arrange in this pattern, but they don't remain spherical – they contort into rhomboids.) (Hardie, *Kepler*, p. 29)

Kepler is describing Bucky's "octet truss" arrangement made from spheres when he writes,

"This arrangement will be more comparable to the octahedron and pyramid."

Kepler adds that if many spheres are placed in a small space, this type of packing,

"will be the tightest possible, so that in no other arrangement could more pellets be stuffed in the same container."

Over the years, this last sentence has come to be known as known as "Kepler's Conjecture." It's his "conjecture" because he theorized about it, but he did not prove it mathematically.

In 1831, the German mathematician **Carl Friedrich Gauss** published a partial solution.

Over a century later, in 1953, the Hungarian mathematician **Laslo Toth** found it could be proven with a finite (but very large) number of calculations. Alas, the power of computer wasn't available then.

In 1992, University of Michigan professor **Thomas Hales** and one of his graduate students, Samuel Ferguson, started writing a computer program testing Toth's finite number of arrangements. In 1998, Hales (now at the University of Pittsburgh) announced that the proof was complete. After four years of inspection, a panel of independent referees are "99% certain" the proof is correct.

Mathematicians call it "Kepler's Conjecture," but it's clear that **Thomas Harriot knew** about the 12-around-1 cuboctahedral arrangement before Kepler.

Sir Walter Raleigh had asked his friend Harriot the most efficient way to stack cannonballs and how much deck area aboard a ship would be required. Harriot didn't need to count cannonballs.

Knowing their diameter, he mathematically calculated how many cannonballs would stack in pyramidal piles with bases that were triangular, square, and rectangular shapes.

(Ralph Staiger, *Thomas Harriot, Science Pioneer*, p. 66, NY Clarion, 1998)

What does this have to do with John Dee?

Here's the connection: **Harriot and Dee were friends**. Even though Dee (1527-1608) was 43 years older than Harriot (1560-1621), Dee lived to be 81, so their lives overlapped. They were both passionate about the same things – mathematics, optics, navigation, the colonization of America, and, of course, atomism.

It's known that they met at least three times. Dee's personal diary shows that he had a meeting with "Mr. Harriot" on August 27, 1592 and again on March 18, 1594. (Remember, Harriot only lived three miles upriver from Dee.)

Earlier, in 1590, the 30-year-old Harriot had presented the 63-year-old Dee with two books: *The Voyage of Antonio de Espejo* and *On the New World*.

The first book was published in Paris by Richard Hakluyt in 1586. In 1582, this Spanish explorer, Antonio de Espejo, had settled a group of villages along the Concho River (where one branch of the Rio Grande starts) in what is now the state of Chihuahua, Mexico. In 1583, he led an expedition into what is now New Mexico and Arizona. This book, now in the British Library, is inscribed,

"Joannes Dee, Ao 1590 Januarij 24 ex dono Thomae Hariot, Amici mei."

(John Dee, January 24, 1590, Given by Thomas Harriot, my Friend)

(Sherman, p. 84 and Roberts and Watson, D8)

On the Title page, Dee has also "written "Ao 1583" (presumably this means Annus (or year) and presumably the Ao stands for "in the year of" as Espejo's voyage was in 1583 (It's ironic that this is the same year that Sir Humprey Gilbert died, causing the settlement on the John Dee River to be abandoned.) It's known that Dee read the book, as it is underlined in places.

John Dee and Thomas Harriot were friends

The second book given to Dee was *On the New World* by Peter Martyr. This Italian born historian was a chaplain and teacher in the Spanish court of Ferdinand and Isabella. He was the "Richard Hakluyt of Spain,"recording copious accounts of the early Spanish expeditions to the New World, from Columbus' voyage to Balboa's discovery of the Pacific Ocean. He was the first to recognize the importance of the Gulf Stream.

Dee also signed this book "John Dee, January 24, 1590; London, given by Thomas Harriot, my friend." (Roberts and Watson, D1)

Dee was an Atomist

Robert Kargon, in his 1966 book *Atomism in England, from Hariot to Newton*, prefaces his chapter on Harriot with a few words about John Dee.

He calls Dee the

"leading participant in the Platonic-Pythagorean
revival of the English Renaissance"
(Kargon, p. 8)

"Dee's number mysticism,
best exemplified by his acceptance
of the dictum that soul is a number moving itself,
was balanced by a strong respect for 'experiment'."
(Kargon p.8 and Dee, *Preface* IV)

Kargon mentions "Dee's pupil, Thomas Digges, with his Copernican outlook, and William Gilbert, who wrote an important treatise on magnets." He also mentions the Italian philosopher Giordano Bruno, who lectured in England.

These four men (Dee, Digges, Gilbert,and Bruno) represent:

"new currents in natural philosophy in England:
Copernicanism, Platonism-Pythagoreanism,
a new emphasis upon experiment, and atomism."
(Kargon, p. 11)

Prejudice against New Ideas (even if they were ancient ideas)

One of the loudest voices against Dee and Harriot was the Jesuit Robert Parsons. Parsons had taught at Oxford, but was forced to emigrate to the continent because of his strong Roman Catholic viewpoint. In his preaching and writings, he advocated the overthrow of Elizabeth I by continental Catholic powers.

In 1592, Parsons wrote a scathing attack (widely circulated throughout England) on Sir Walter Raleigh and the Northumberland circle, which has been called "The School of Night."

"Of Sir VValter Rawley's Schoole of Atheisme by the waye,
and of the Conjurer that is M. therof,
and of the diligence vsed to get young gentlemen to this schoole,
where in both Moyses, and our savior, the olde, and the new Testamente are jested at,
and the schollers taught amonge other thinges, to spell God backwarde."

The "M" refers to Master. As biographer John Shipley sees it: "Both John Dee and Harriot considered that Parsons was referring to themselves, and they quite possibly may have discussed the matter when they met at Dee's home at Mortlake shortly after the publication."

Ernest Strathman, in *John Dee as Ralegh's Conjurer,* shows evidence that Dee was called "Master of the School of Atheism." Quinn and Shirley provide evidence that Harriot acknowledged himself to be the Master. So, this is something else they had in common: both of them were accused and slandered for expressing their opinions.

(John Shirley, *Thomas Harriot: A Biography,* p. 180, *A Contemporary List,* p. 21*)*

In the profoundly religious 1500's, ideas like atomism and the camera obscura, no matter how scientifically or mathematically sound, first had to become acceptable theologically. And with so much internal debate within Christianity, "new" ideas took a long time being judged.

In the interim, people espousing these new ideas became targets for theological outrage. Dee was bubbling to share his knowledge with the world, so he veiled in alchemical language of the Monas. Harriot decided just to keep mum.

O.K., so Dee was an atomist and a friend of Thomas Harriot in the 1590's. That still doesn't prove he knew about closest packing of spheres 25 years earlier, in 1564.

A "subtle" clue that Dee knew about the 12-around-1 closest packing of spheres arrangement

Surfing the web for closest packing spheres, I came across reference to an article entitled *"An Unconventional View of Closest Sphere Packings"* by a Swiss crystallographer G. O. Brunner. The word "unconventional" sounded interesting, so I journeyed deep into the 7th floor stacks of the Brown Science Library to find the article in the July 1971 copy of *Acta Crystallographica*.

Unfortunately, it turned out to be a technical paper for advanced crystallographers involving things like "cation repulsion" and "Coulomb's Law." But, right at the end was a small section entitled "Remarks and historical details." There was a reference that I had not seen in any other text on closest packing of spheres. It read:

Kepler (1611) and Harriot (1560-1621)
are frequently referred to as the earliest references;
the coordination number 12 of the closest packings,
however, was already mentioned by Cardanus (1550).

(Brunner, p. 390)

Cardano's "1550" book is *De Subtilitate* or *On Subtleties* [of natural phenomena]. This is the same book in which Cardano explains the benefit of using a lens in a camera obscura.)

So, I trekked across the Brown Green through a sea of white chairs set for graduation ceremonies, to the venerable, white-marble John Hay Library, where all the old books live.

They had the five-volume set of Cardano's *Opera Omnia*, (*Complete Works*), published in 1663. The librarian delivered the huge leather-bound volume to me in the high ceiling reading room. Marble busts of early scholars looked over my shoulder from their perches high atop the old wooden bookshelves.

I am not particularly fluent in Latin but scanned the page, which G. O. Brunner had cited for words I might recognize. There, in the second column of text, was the only number on the entire page: the number 12. It was in a sentence which read:

**"sed cur quae ab aliis circumdantur, funt
hexagma forma, cum fphaera à 12."**

Here was something about "**circular,**" **hexagon formed**," and "*as a sphere has 12*." The librarian provided me with a copy which I brought to my translator Peter Lech, then a Grad student in Brown's Classics Department. Here's what Cardano wrote (bold emphasis is mine):

"But why the crystal has six faces
(for it hardly ever has more or less)
must now be explained.

The reason is that,
just as the individual cells of bees are surrounded by other cells,
and for that reason are themselves hexagonal,
so individual pieces of crystal are surrounded by others.

But why are things which are surrounded by others, of hexagonal shape
when **a sphere is surrounded by 12 similar spheres**, not by six?

Thus, it is better to ascribe this number to the nature of the body.

For an entire body that is surrounded by rectilinear surfaces,
is distinguished in respect to length, width, and height.

This corporeal nature consists of six opposed faces;
and for this reason crystals and the other gems of this sort,
like the beryl, have six faces each."

Dee had 20 books written by Cardano in his library. Two of them were copies of *De Subtilitate* (the first edition of 1550 and the second edition of 1554). Dee even took the 1550 edition with him on his trip to Prague in 1583.

Dee and Cardano (1501-1576) were both brilliant polymaths, curious and knowledgeable about all the sciences and arts, from astrology to zoology.

One special interest, which is obvious in the text *Monas Hieroglyphica,* and discussed at length in *De Subtilitate,* is cryptology.

(Galland, p. 34 in http://home.hiwaay.net/-paul/cryptology/history.html)

"12 spheres around 1"
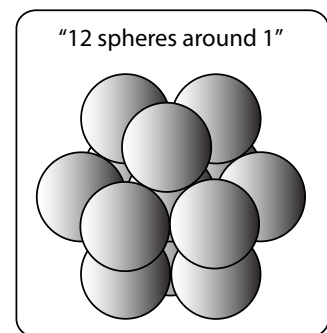
A brief biography of Cardano

The title of Cardano's great work d*e Subtilitate* literally translates as *On Subtlety*. Nowadays, "subtle" means "hard to detect" or "not immediately obvious," but in its Latin origins, it meant "delicate, fine," which became "**crafty or ingenious.**" (Oxford American Dictionary, p. 1697)

Cardano's publisher, Johannes Petreius from Nuremberg,
summarized the book on the Title page:

**"The causes, powers, and properties of more than 1500
varied, uncommon, difficult, hidden, and beautiful things,
all of them observed by the author, in various places, by personal trials.**
(Petreius, in Cardano, in Grafton, p. 163)

There's a strange reason why not many people are familiar with Giralmo Cardono. **He was too prolific**. When his complete works (*Opera Omnia)* were published after he died, they totalled over 7000 pages and had to be separated into 10 books. As his biographer Anthony Grafton writes in *Cardano's Cosmos: The World and Works of a Renaissance Astrologer,* the length, variety, and technical density of Cardano's books have deterred scholars from approaching him."

Cardano's father, Fazio Cardano, was such an expert mathematician that **Leonardo da Vinci** consulted him on questions about geometry. Fazio prepared **John Peckham's** *Common Perspective* to be printed for the first time. (Peckham's manuscript discussed the eye, vision, refraction, mirrors, and, of course, the camera obscura.) This text was a major influence on scholars like da Vinci, Maurolico, della Porta, Snell, and Kepler.

Young Giralmo Cardano was one of the brightest students at Padua University. Later, as a teacher and doctor, he started producing books on philosophy, theology, medicine, astronomy, and mathematics.

In 1540, at age 44, he published a groundbreaking work on algebra, *Ars Magna*. In it, he demonstrated how to solve cubic and quartic equations. His books became best sellers, not just in Italy, but all across Europe. Often, his works were pirated by publishers in far away cities.

In 1552, at age 51, he was offered over 2000 gold crowns to journey to Scotland and treat the ailing Archbishop of Saint Andrews. As he traveled through the major cities of Europe, he was greeted **"as a celebrity and the world's leading scientist."**

Cardano was in Scotland for less than three months but managed to cure the Archbishop. But, while he was in England, guess who he met with?

Cardano meets Dee in 1552

Grafton writes, "A common interest in medical astrology evidently brought Dee and Cardano together." Both were interested in celestially powered stones like one that Marcello Ficino wrote of in *De Vita*:

**"I saw a stone at Florence, brought from India,
where it had been dug from the head of a dragon.
It was round, in the form of a coin, and naturally and neatly decorated
with a great many points, which were rather like stars."**
(Ficino, in Grafton, p. 112)

Dee, of course, had Ficinos' De Vita in his library (several copies).
In one copy, Dee wrote in the margin next to Ficino's story of the Indian rock:

"I say a stone like this one, of the same quality in 1552 or 1553.
Present were Cardano of Milan, John Francis [Cheke]
and Monsieur Braudaulphin, the ambassador of the French King
in the ambassador's house at Southwork.

When vinegar was poured on it, it moved a little way
in a straight line, then to the side,
and soon began to move in a circle,
until the vapor of the vinegar disappeared."

(Ficino in Grafton, p. 112, Ficino, *De Vita coelitus comparanda*, Venice 1516, folio 160)

Grafton points out that "gems capable of drawing down the power of particular planets" were prized by "consumerist elite of Medicean Florence." He adds, "Cardano eagerly collected celestially powered stones like these," or at least tales of them, which he recounts in *De Subtilitate*.

In a sense, Dee had paved the way for Cardano's acceptance in England.
Grafton explains that Cardano,

"was traveling the twisting, complicated corridors
of an existing political and social system–
one in which entrepreneurs of power over nature like John Dee
had already established a position
as advisers to men of power over the state."

(Grafton, p. 111)

Echoing William Sherman's conclusions in *John Dee: The Politics of Reading and Writing in the English Renaissance*, Grafton adds:

"Despite many legends to the contrary,
Dee and his patrons were not obsessed with occult ways of knowledge.

Dee belonged at least as much to the Renaissance tradition
of practical engineering and navigation
as to that of scrying and natural magic,
and offered Queen Elizabeth and William Cecil
practical advice and technical knowledge of many kinds."

(Grafton, pp. 1111-112)

To summarize, both Kepler and Cardano wrote about "12-spheres around 1 close-packing." I have shown a strong connection between "Kepler, Harriot, and Dee" and another between "Cardano and Dee."

Dee knew about closest packing of spheres. He was so amazed by it and its connection to (as we shall see) mathematics and optics, he wanted to let the world know. To proclaim what he knew, and still be selective about who found out, he disguised his story in the cryptic *Monas Hieroglyphica*.

How can we be sure Dee knew about the cuboctahedral shape made from lines? (the 24 edges)
The answer to this is a lot easier.

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Dee knew about the cuboctahedron

Dee's involvement in the 1570 translation of Euclid's Elements is proof he was well aware of the cuboctahedron, Bucky's vector equilibrium. The proof can be found in a "brief treatise" by Flussas that was appended after "Book 16," at the very end of the long work.

(At the bottom of the Title page is a figure of that changeable character Mercury, along with a panoply of classical astronomers and the quadrivium, or the 4 Liberal Arts: Geometry, Arithmetic, Astronomy and Music.)
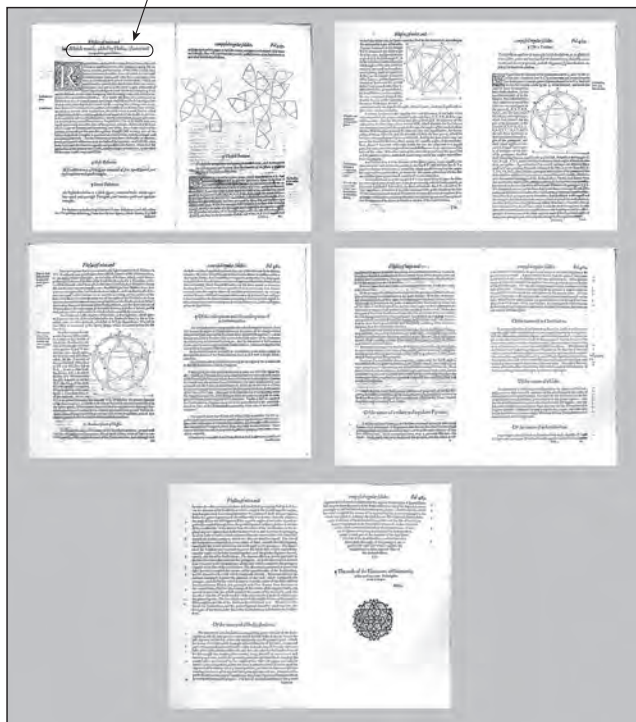
The Title page of Henry Billingsley's (and John Dee's) Elements of Euclid

A briefe treatise, added by Flussas, of mixt and composed regular solides.

Flussas' "brief treatise" takes up the last 10 pages of the text that is almost 1000 pages long.

The intersections of the various "duals" among the Platonic solids
are either Archimedean solids or Platonic solids.

"Regular solids are said to be composed and mixed,
when each of them is transformed into other solids,
keeping still the form, number, and the inclination of the bases,
which they before had, one to the other.
Some are transformed into mixed solids,
and other others into simple solids."

What Flussas is referring to by the term "**mixed**" solids
are the Archimedean solids, irregular polyhedra with 2 different types of faces.
And "**simple**" solids" are the Platonic solids, the 5 regular polyhedra

The intersection of a cube and an octahedron
is a cuboctahedron,
an Archimedian solid.

"If you divide the lines of a Cube and an Octahedron into two equal parts,
and couple the sections,
the solid angles subtended of the plane superficies
made by the coupling lines being taken away,
there shall be left a solid, which is called an Exoctahedron."

"First definition:
An Exoctahedron is a solid figure
containing six equal squares
and eight and equilateral and equal triangles."

Exoctahedron
is Flussas' term for a **cuboctahedron**
(or Bucky's **vector equilibrium**).
They all refer the exact same thing.

The " Exo" part is short for hexahedron,
another name for a cube
(having 6, or hex, sides).
The silent "h" got dropped.

The intersection...

...of a cube...

...and an octahedron...

...is a cuboctahedron.
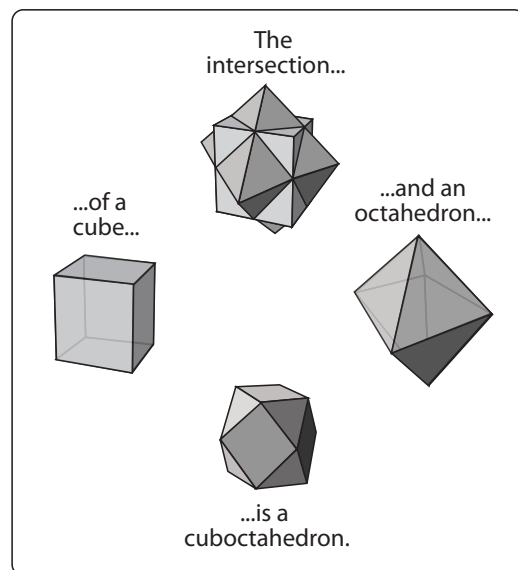
The intersection of a dodecahedron and an icosahedron
is an icosidodecahedron,
another Archimedian solid.

"A Dodecahedron and an Icosahedron
are transformed or altered into mixed solid.

If you divide their sides into two equal parts,
and take away the solid angles subtended of the plain superficial figures
made by the lines coupling those middle sections,
the solid remaining after the taking away of those solid angles,
is called an Icosidodecahedron."

"Second definition:
An Icosidodecahedron is a solid figure containing
12 equilateral, equal, and equiangle Pentagons,
and 20 equal and equilateral triangles."

The intersection...

... and a
dodecahedron...

...of an
icosahedron...

...is an
icosidodecahedron.
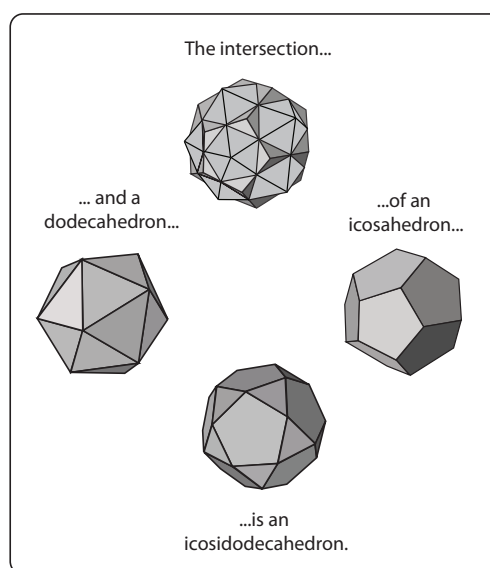
But, the intersection of 2 tetrahedra is an octahedron,
a <u>Platonic</u> solid

"The solid which is made from a
Dodecahedron and an Icosahedron
shall be called and Icosidodecahedron.
And likewise, the solid made of a Cube and an Octahedron,
shall be called and Exoctahedron.
But the other solid, namely a Pyramid (or Tetrahedron)
is transformed into a simple solid."

"For if you divide into two equal parts
every one of the sides of the Pyramid,
triangles described by these lines which couple the sections,
and subtending,
and taking away solid angles of the Pyramid,
are equal and like unto the equilateral triangles
left in every one of the bases,
all these triangles produce an Octahedron,
which is a simple, and not a composed solid."

"For the Octahedron has four bases,
alike in number, form, and mutual inclination
with the bases of the pyramid,
and has the other four bases with a like situation,
opposite and parallel to the former.

Therefore, the application of the Pyramid taken twice,
makes a simple Octahedron,
as the other solids make a mixed compound solid."

The intersection...

...of a tetrahedron...

...and another tetrahedron ...

...is a small octahedron.
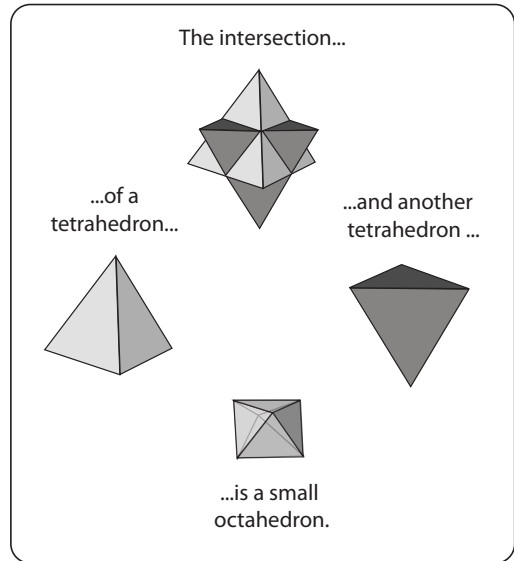
Like Bucky, Flussas has recognized that the tetrahedron is more special than the other 5 Platonic solids. It's unique not only because only is it the only self-dual, but also because its "intersection" with itself is another Platonic solid.

This octahedron is the one we found in the middle of the "giant tetrahedron" made from 4 smaller tetrahedra.
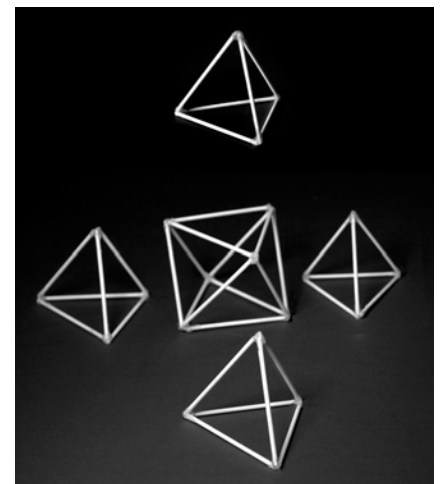
Boiled down even further, this is the octahedron that shares a face with a small tetrahedron – the essence of Bucky's octet truss!

Bucky's illustration for his octet truss patent

120
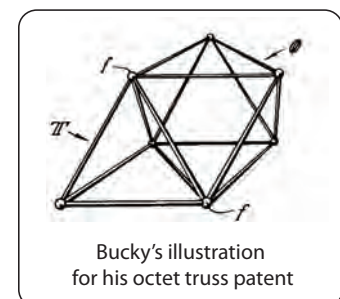
"The first problem.
To describe an equilateraland equiangular exoctahedron,
and to contain it in a sphere given.
And to prove that the diameter of the sphere
is double to the side of the said octahedron."

A cuboctahedron
contained within a sphere

The diameter of such a sphere is essentially any two radiating vectors of a cuboctahedron that are in a straight line. So, half of the diameter is one radiating vector.

In the cuboctahedron,
each of the 12 radiating vectors
is the same length
as each of the 24 edges.

length of edge vector
length of radiating vector

1.6329931619	tetrahedron	
1.4142135624	octahedron	5 "Platonic Solids"
1.0514622242	icosahedron	
1.1547005384	cube	
0.7136441795	dodecahedron	
1.0000000000	**cuboctahedron**	
0.6180339887	icosidodecahedron	
0.8528028654	truncated tetrahedron	
0.5621692754	truncated cuboctahedron	
0.6324555320	truncated octahedron	13 "Archimedean Solids"
0.3367628118	truncated dodecahedron	
0.4035482123	truncated icosahedron	
0.7148134887	rhombicuboctahedron	
0.4314788105	great rhombicuboctahedron	
0.4478379596	rhombicosidodecahedron	
0.2629921751	great rhombicosidodecahedron	
0.7442063312	snub cube	
0.4638568806	snub dodecahedron	

Thus, Flussas is proving that a radiating vector and an edge are the same length.

Remember, this is the characteristic that makes the cuboctahedron so special. It's why Bucky called it a vector equilibrium.

It's worth repeating this chart which shows that the cuboctahedron is the **only one** of all the Archimedean or Platonic solids whose radiating vectors and edges are the same length.
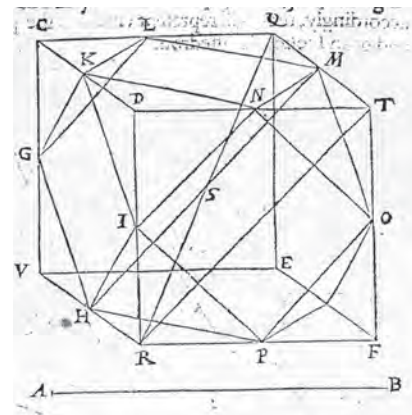
Flussas elaborates with into a
one-and-a-half-page-long geometric proof
based on this drawing, which is essentially
a cube with the corners clipped off.
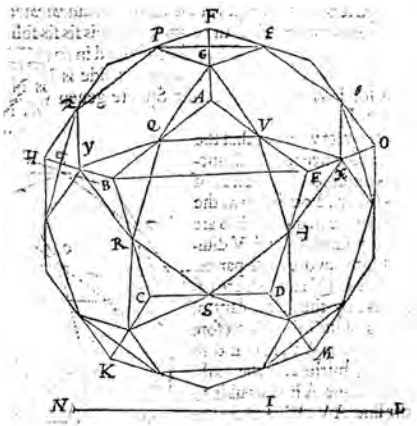
Flussas then poses several "problems" which he "proves geometrically. He also proves a "problem" about the icosidodecahedron using this geometric drawing.

He also discusses the "inscriptions and the circumscriptions of the icosidodecahedron." (In other words, how the icosidodecahedron can be contained by any of the five Platonic solids, and how any of the five Platonic solids can be contained within it.)

He also provides flattened versions of the cuboctahedron and the icosidodecahedron that can be cut out and folded into small 3-D models.

"For the better understanding of the two former definitions, and also of the two Propositions following, I have here set two figures, whose forms, if you first describe on pasted paper or such like matter, and then cut and fold them accordingly, will represent unto you the perfect forms of an Exoctahedron and of an Icosidodecahedron."

Flussas' fold-up "exoctahedron" or cuboctahedron
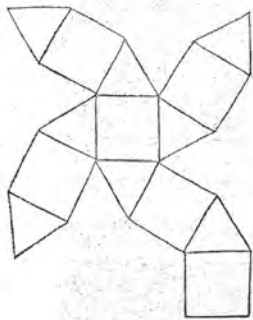
Flussas' fold-up icosidodecahedron
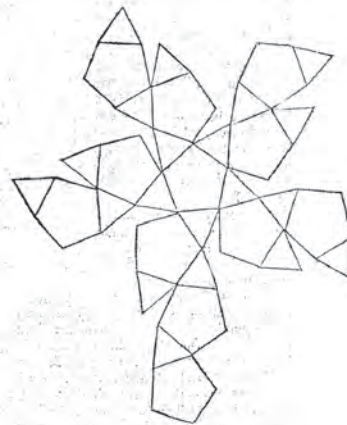
Of the nature of a trilater and equilater Pyramis.

Of the nature of an Octohedron.

Of the nature of a Cube.

Of the nature of an Icofahedron.

Of the nature of a Dodecahedron.

Finally, he provides more details about the various characteristics of the five Platonic solids.

As Flussas' "Brief Treatise"
is the at the very end this work
(of 495 double page folios),
it is followed by the following sentence:

Under this is a decorative element which is
repeated occasionally throughout the text.
It is not hard to see the two intersecting
equilateral triangles, the skeleton of
the 6-around-1 pattern
of closest-packing-of-circles.
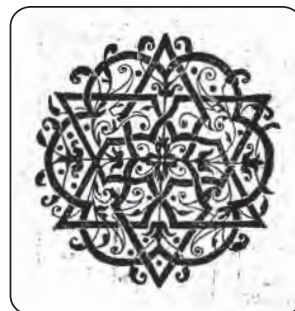
So it's pretty obvious that Dee knew all about the cuboctahedron. This is not simply a book that the owned, he was instrumental in its production and wrote its astute *Preface*.

One might suggest it was Henry Billingsley who was responsible for adding Flussas' "Special Treatise." However, a close study of the text indicates that **it was Dee who was responsible for this addition.**

To understand why I can make this claim, the text must first be put in its proper historical perspective.

Here's how Carl Boyer puts it in his 1968 *History of that Mathematics*:

**"The *Elements* of Euclid not only was the
earliest major Greek mathematical work to come down to us,
but also the most influential textbook of all times."**
(Boyer, p. 119)

Euclid of Alexandria wrote the ***Elements*** around 300 BC. Proclus claims that Euclid had compiled the wisdom of other Greek mathematicians like Eudoxus and Theaetetus. Regardless, it was Euclid who organized this wisdom into clear and logical demonstrations.

Roman scholars, like Cicero, read this text. Boethius translated it into Latin around 580 AD. Harun al Rashid translated it into Arabic around 800 AD.

The Italian mathematician Giovanni Campano (1220-1296) made his own translation into Latin from the original Greek. An edition of Campano's translation, printed in Venice in 1482, makes it one of the earliest math books ever published.

Since then, over 1000 different editions of this work have been printed
in numerous languages. As Boyer declares:

**"perhaps no book other than the Bible can boast so many editions,
and certainly no mathematical work has had an influence
comparable with that of Euclid's Elements."**
(Boyer, P. 119)

For centuries, it was required reading for all university students. It influenced the thinking of Copernicus, Kepler, Galileo, and Newton. The reason you might not have read this all-time bestseller is that in the 1900's, its content was absorbed into the geometry textbooks you studied in high school.

A brief outline of some of the
various translations of Euclid done in Dee's era.

1545 **Petrus Ramus** (1515-1572), who has been called the most eminent logician of his time, published a commentary of *Elements* in 1545 and again in 1549. Dee not only owned the 1549 text, but had met with Petrus Ramus in Paris.   (Roberts and Watson, number 948)

1551 **Oronce Finé** (pronounced "Fee-nay, 1494-1555) wrote a commentary on the first 6 books of Euclid. Dee not only owned this text, and 10 others, by Finé, but they were friends and they corresponded over the years about mathematics and navigation.   (Roberts and Watson, numbers 460, B252)

1557 **Jean Pena**, (ca. 1500–1558), royal professor of mathematics in Paris, translated Euclid's *Optics* and *Catoptric*s (Mirrors) into Latin. Dee knew Pena and owned 2 copies of his translations.   (Roberts and Watson, numbes 373, 1866)

1557 **Jacques Peletier** (known as Peletarius, 1517-1582) published six books containing "*Demonstrations to the geometrical Elements of Euclid*," showing the Greek, his Latin translation, as well as his comments. Dee owned Peletier's book as well as several of Peletier's other books on algebra.   (Roberts and Watson, numbers 80, 260, 475, 532, 1119)

1559 **Johannes Buteo** (1492-1572) published in his book *De quadratura circuli* (On squaring the circle). The appendix contains notes "on the errors of Campanus, Zambertus, Orontius (Oronce Finé) Peletarius, Pena, who were all interpreters of Euclid. Dee owned this book by Buteo as well as several of Buteo's other works on arithmetic and logic .   (Roberts and Watson, numbers 347, 961, 963)

1565 An Italian translation was published in 1565 by **Nicollo Tartaglia** of Venice (1499–1557).

1566 **Franciscus Flussas Candalla**   (Françoix de Foix, Compte de Candale)   (1502–1595) "restored" the first 15 books of Euclid. This French mathematician adopted the nickname "Flussas," which may be an amalgam of his name, François de Foix, and the Latin word "fluxus" meaning "flowing." (Roberts and Watson, number 18)

Foix (pronounced "Fwa") is a small province in southern France, just across the Pyrenees from Spain.

Compte de Candale is a title in the French aristocracy, which, strangely enough, derives from the town of Kendal in the Lakes District of northern England. It is famous for its manufacturing of green cloth, worn by Robin Hood and his Merry Men.

Nearby Scotland had a connection with the French aristocracy for generations. In 1462, the Earl of Kendal gave his allegiance to the King of France. It wasn't long before he had to move to France, but he and his descendents continued to style themselves le "Compte de Candale," the "Count of Kendal."

So, one might translate this gentleman's name as Frank "the Flow" Kendal (Frank from Foix, the Count of Kendal). Dee simply referred to him as Flussas.

1570 **Henry Billingsley** (d.1606) (and **John Dee,** 1527-1608) publish Euclid's *Elements* for the first time in the English language.

1572 **Federico Commandino** (1509-1575) the Italian polymath, made a translation of *Elements* that followed the Greek more closely than others. Dee visited Commandino in Urbano, Italy. They were friends and collaborated on publishing Machometus Bagdedinus' *De superficierum divisionibus* (Mohammed of Baghdad's *On the division of the surfaces*).

> Incidentally, all the translations done in this era credited
> "Euclid of Megara" as the author of Elements. This Euclid was
> a contemporary of Plato, living around 400 BC. The real author
> was actually a different person, "Euclid of Alexandria" who
> lived a century later, around 300 B.C. (Heath, Vol. 1 p.3)

After Henry Billingsley graduated from Dee's alma mater, St. John's College in Cambridge, he apprenticed with a London haberdasher and soon became wealthy merchant. (His date of birth is not known, but as he died about the same time as Dee, it seems as tough the two might have been college chums. Dee would have been eager to collaboate with him on this ambitious project not only because their shared love of Euclid, because of Billingsley's deeper pockets.)

Twenty-five years after the publication of *Elements*, he became Lord Mayor of London and was knighted as Sir Henry Billingsley. In his introduction to the translation of *Elements* he proposed, "to translate some other good authors, … pertaining to the Mathematical Artes," but he never did.

Dee, on the other hand, was somewhat renowned for his knowledge of Euclid. His library contained **over 25 different translations** of *Elements*. He also had many commentaries on *Elements*, as well as Euclid's other texts *On Optics, On Catoptrics and On Perspective*.

When he was only 24, Dee lectured in Paris on Euclid's *Elements*, as he recounts in his autobiographical 1592 *Compendious Rehearsal*:

> "From Louvain, I took my journey towards Paris on the 15th day of July, in the year 1550, and came to Paris on the 20th day of that month. Later, within a few days after, (at the request of some English gentleman, who suggested I do it for the honor of my country), I did undertake to read freely and publicly Euclid's Elements Geometrical, *Mathematicé, Physicéand Pythagoricé*, a thing never done publicly in any University of Christendom. My audience at Rhemes College was so great and for the most part older than myself, that the mathematical schools could not hold them. Many were happy just to be able to see or hear the lecture through the school's windows, as best they could.
> I did also speak on every proposition, besides the **first exposition**. **And by the first four principal definitions** [emphasis mine], representing to the eyes (which by imagination only are exactly to be conceived) a greater wonder arose among the beholders than of my Aristophanes' *Scarabeus* mounting up to the top of Trinity-hall in Cambridge (as mentioned above). Of this mathematical reading, very many testimonies lie here before you."

Dee continues:

"In that University of Paris, were at that time over 40,000 accounted students; some out of every quarter of Christendom being there. Among these, very many of all estates and professions were desirous of my acquaintance and conference, like Orontius, Mizaldus, Petrus Montaureus, Ranconetus, Danesius, Jacobus Sylvius, Jacobus Groupylus, Turnebus, Straselius, Vicomercatus, Paschasius Hamelius, Petrus Ramus, Gulielmus Postellus, Fernelius, Jo. Magnionus, Johannes a Pena, & etc., as some of their letters lying on this table attest.

There, I refused to be one of the French King's mathematical readers, with 200 French crowns yearly stipend offered to me, if I would stay for it. I likewise refused a good stipend of Monsieur Babeu; and better than that, of Monsieur de Rohan; and better than that of Monsieur de Monluc, who was then sent as ambassador to the Great Turk.

And not only in Louvain and Paris Universities has God sent me good credit and estimation with a favor and love of very many (noble lovers of good learning, or well learned themselves), but also in Orleans, Cologne, Heidelberg, Strasburg, Verona, Padua, Ferrara, Bologna, Urbino, Rome, and (to conclude) in many other universities, cities, and towns of Christendom; as may appear by the multitude of letters and other records lying here to be seen and perused in this case; from the year 1547 until, and in, this present year of 1592."

Needless to say, Dee was a pretty well-traveled and well-connected. During his a Parisian lecture on Euclid, he claims to have spoken on every proposition. As there are over 450 propositions. That's a lot of explaining.

He also gave some kind of a demonstration on the "first 4 principal definitions" which are:

1   A point is that which has no part (is not able to be divided)
2   A line is a length without breadth.
3   The extremities of a line are points.
4   A straight line is a line which lies evenly with the points on itself.

What kind of demonstration could he have given of these four definitions that is as dramatic as his huge Scarab Beetle, with a person on it, flying up to the rafters in his college play?

Dee leaves a clue by calling his commentary
on these these definitions the "first exposition."

Book 1 of Euclid starts with 35 definitions. The last sentence of the paragraph introducing these definitions (in the 1570 translation of Euclid) reads:

"And forasmuch and as all the demonstrations and proofs
of all of the propositions in this whole book
depend upon the following grounds and principles…
here are set certain short and manifest expositions."

Dee's use of the word "**exposition**" in his 1592 autobiographical text **and** in this introductory paragraph of the 1570 translation of Euclid is a pretty good clue that the explanatory matter following the definitions was written by Dee.

But there is more evidence. The phrasing, sentence structure, content and in some cases the actual words match Dee's "Preface" found at the beginning of this work

For example, in Definition **1,** "**A point is that which has no part**," the exposition also gives the "Pythagorean" definition, "**A point is a unity which has no position**."
 The author challenges this definition, saying "Unity" is part of "Number" and

"numbers are conceived of the mind...and are without
place and position... For a point is material, and requires
position and place, and is thereby different from unity"

Dee says the exact same thing in his *Preface* of this very book (page a.ij):

"A point may have a certain determined Situation… here, there, yonder & etc.
Herein (behold) our Unit is free and can abide no bondage, or be tied to any place…"

The explanation for Definition **3** says "**although unity be the beginning of numbers, and no number.**" This echoes Dee's sentence in the "Preface" "**We account a Unit, a thing Mathematical, though it be no Number**." (Preface, p.j.)

For the "exposition" of Definition **4**, "**A straight line is that which line of equally between its points**," the author finds 8 similar explanations of a line, ranging from those of Plato and Archimedes to Campanus' 1482 translation. Few people would have 8 definitions from 8 different authors at their fingertips. Dee would, but a London haberdasher wouldn't.

But it's the "exposition" for Definition **2**, "**A line is a line without breath**," gives us a clue about the Dee's dramatic demonstration that dazzled the Parisians.

Dee provides an alternative another definition: "**A line is the moving of a point, as the motion or draught of a pinne or penne to your sense makes a line**."

The *Oxford English Dictionary* provided a clue as to what this means. A pennon is a long, narrow flag or streamer which a knight would attach to the tip of his lance.
To one's eyes, the motion or flow (draught) of a streamer makes a line.

Basically, Dee was doing a demonstration of the young girls' Olympic gymnastic event that involves swirling small wands with ribbons attached.
The tip of the wand represented a point, and the line follows it's path. One can imagine Dee twirling the pennon in a large circular pattern or a figure-8 pattern in the front of the crowd of mathematicians, visually demonstrating that a line is a point in motion.

This is pure Dee. In Theorem 7 of the Monas, Dee writes, "geometricians teach that a line is produced by the flowing of a point… Lines… are like DROPS (like physical points) that continuously Fall (as if FLOWING)…" (Dee also illustrates this idea in the Vessels of the Holy Art diagram in Theorem 22 of the *Monas*)

Besides these 35 definitions, just how much of the commentary on Books 1–9 (which involve 2-D or plane geometry) belongs to Dee and how much belongs to Billingsley is debatable. It is known that Billingsley was also a classical scholar.

But the **commentary** for Books 10-13 (involving 3-D shapes or spatial geometry) is clearly Dee's work. Dee even tells us in his 1592 *Compendius Rehearsal* that he had done "diverse many Annotations and Inventions dispersed and added in and after the tenth book of the aforesaid English Euclid in the year 1570" (Dee, Compendious Rehearsal, p. 24, Number 6)

Books 10, 11, 12, and 13, contain not only the translation of Euclid's words, but over 275 short commentaries described as either "demonstrations," "constructions," "problems," "corollaries," or "annotations."

<div align="center">

**Of these 275 supplemental ideas, 120 of them are Dee's
and 155 of them are credited to Flussas.**

(There are also 6 commentaries by Theon and 12 by Campanus.)

But not one was made by Billingsley!

(The author of each particular commentary is clearly noted,
but the various commentaries are quite intermingled).

</div>
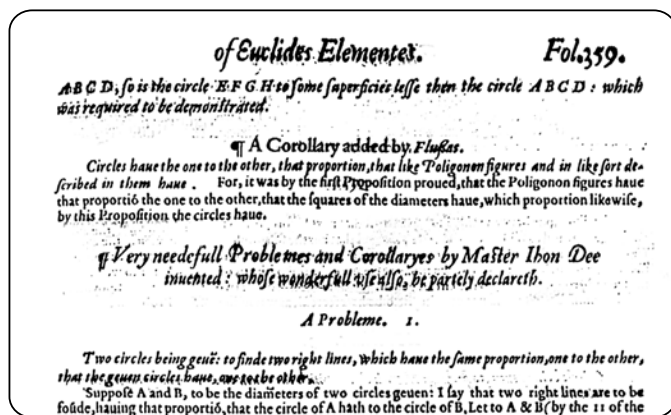
For example, on page 359,
there is a corollary by Flussas,
followed by some
" Very needfull
Problems and Corrollaries
invented by Master John Dee:
whose wonderful uses
he partially declareth."

And on page 381 verso,
there is a corollary by Flussas,
followed by a corollary by Dee,
followed by "Certain Theorems and Problems
(whose use is manifold, in Spheres,
Cones, Cylinders, and other solids)
added by John Dee."
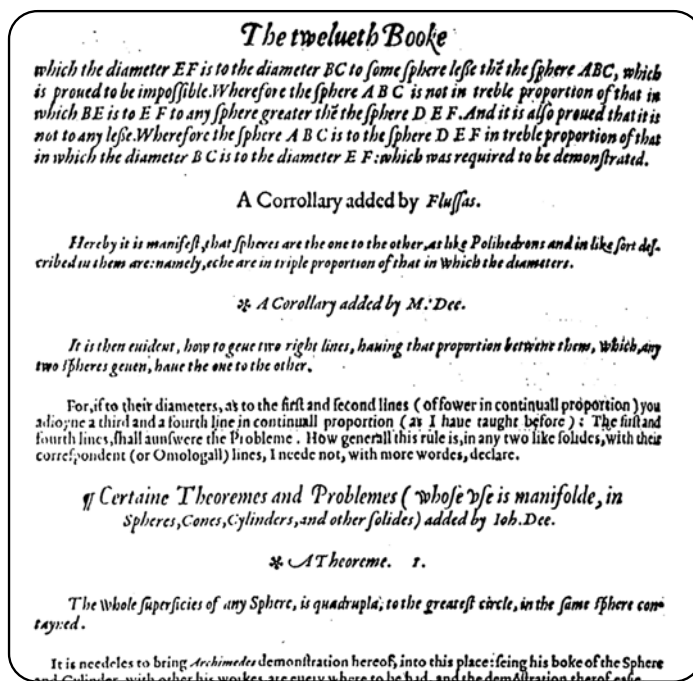
Scholars feel that book 14 and 15 are so different from Books 1–13 that they are "apocryphal," meaning they were not written Euclid.

They credit the Greek mathematician Hypsicles (ca. 190 BC–ca. 120 BC) with writing Book 14, based on a treatise by Apollonius (ca. 262 BC–190 BC) who has been nicknamed "The Great Geometer," based on his treatise entitled *Conics*. Book 14 is a continuation of Euclid's work on how the various Platonic solids fit inside spheres.

Book 15 is thought to have been written much later, perhaps by Isadore of Miletus (ca. 480 AD–ca. 550 AD). It explains how each of the Platonic solids fits inside each of the other Platonic solids.

perfect, and shewed it unto them, whereas they made reuysed. By which wordes it semeth to be manifest that *Apollonius* was the first author of this booke, which was afterward set forth by *Hypsicles*. For so his owne wordes after in the same preface seme to import.

❧ The Preface of *Hypsicles* before
the fourtenth booke.

Rend Protarchus, who that Basilides of Tire came into Alexandria, hauing familiar frendship with my father by reason of his knowledge in the mathematicall sciences, he remayned with him a long time, yea euen all the time of the pestilence. And sometime reasoning betwene themselues of that which Apollonius had written touching the comparison of a dodecahedron and of an Icosahedron inscribed in one and the selfe same sphere, what proportion such bodies haue the one to the other, they iudged that Apollonius had somewhat erred therein . Wherefore they (as my father declared vnto me) diligently weighing it, wrote it perfectly. Howbeit afterward I happened to finde an other booke written of Apollonius, which contayned in it the

Book 16, written by Flussas is also included. It has 37 propositions regarding the proportions of the size of the Platonic solids.

They're quite intricate, as you can see by Proposition 32:

> "The proportion of the solid of an Icosahedron to the solid of a Dodecahedron inscribed in it consists of the proportion of this side of the Icosahedron to the side of the Cube contained in the same sphere, and of the proportion tripled of the diameter which connects the centers of the opposite bases of the Icosahedron."

Most modern translations of Euclid only include Books 1 – 13. They don't even include Books 14 and 15, never mind Flussas' Book 16.

But, following Book 16 is that "brief treatise" by Flussas on the intersection of the various Platonic solids making the exoctahedron (cuboctahedron), the icosidodecahedron, and octahedron (made from two intersecting tetrahedron).

A briefe treatise, added by Flussas, of mixt and composed regular solides.

The fact that Dee's and Flussas' commentaries on Books 10–13 are so interwoven, and that that Flussas' Book 16 is so intricate and advanced, leads me to believe that Dee was responsible for appending Flussas' "Brief Treatise" that follows Book 16.

In summary, Dee was responsible for the *Preface* and the important Definitions in Book 1. Billingsley was responsible for Books 1–9. And Dee was responsible for Books 10–15 and for appending Flussas' Book 16 and Flussas' "brief treatise."

Dee held Flussas' translation of Euclid's Elements in high esteem.

This can be seen by studying the first page of Dee's 136-page Library Catalogue of 1583. It is the 18th book listed, (out of 2292 books in total), sandwiched between two books containing the full works of Aristotle (numbers 17 and 19) (Roberts and Watson, p.80)

It's halfway between the *Complete Works of Plato* (the 12th book listed) and Ovid's *Metamorphosis* (the 24th book listed).

The first 2 books on Dee's list of books are 2 *Concordances of the Bible* (alphabetical lists of important words with citations to the passages in which they appear). The next two books are Greek dictionaries, one being the classic "*Suidas*," a dictionary/encyclopedia compiled around 950 AD which has over 30,000 entries.

The point here is that Fussas' translation of Euclid was probably on what I call Dee's "VIB" (Very Important Book) shelf. Dee notes that he took Flussas' translation of Euclid with him on his six-year journey through Europe from 1583–1589.

My conclusion is that it was Dee who recommended appending Flussas' "brief treatise" to the first English translation of Euclid. Thus, Dee was clearly aware of the beauty of the cuboctahedron.

We have also seen that Dee knew that 12 spheres fit perfectly around 1 sphere forming a cuboctahedral shape. Bucky and Dee were both excited about the same thing!

They both had discovered "Nature's operating system."

The Monas Symbol equals the Cuboctahedron

To briefly summarize, Bucky saw cuboctahedron
(his vector equilibrium) as Nature's operating system.

Four pairs of opposing tetrahedra
in the continuous process
of expanding radially outwards
and contracting radially inwards
to the point of vanishment (the 9th thing).
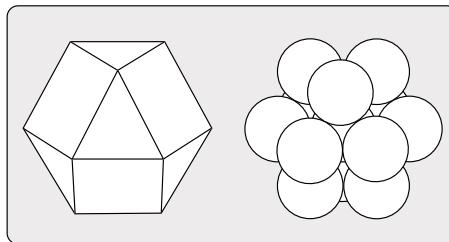
This configuration can be seen as a skeleton of 12 radiating vectors
(which externally connect with 24 edges of the same length.)

Or, it can be seen as 12 spheres around 1, the closest packing of spheres
arrangement that always makes a cuboctahedral shape

Dee recognized the exact same thing 400 years earlier. He knew about the 8 tetrahedra and saw the common centerpoint as 9.

The fact that he appended Flussas' *Brief Treatise* in the first English translation of Euclid's Elements shows he knew about the cuboctahedron. His inclusion of the numbers 12, 13, 24, and 25 in his summary charts shows he understood the cuboctahedron in its 3 different guises:

1. as vectors – 12 radiating vectors connected by 24 edges
2. as spheres – 12 spheres-around-1 makes 13 spheres
3. as a spherical cuboctahedron – which has 25 great circles

In the letter to Maximillian,
he says that those who have studied "**Plenum**" and"**Void,**"
have seen,

**"that the Surfaces of Elements,
which are in close proximity
are coordinated, connected, and Joined Together
by a Law
(decreed by God Almighty)
and Bond
(practically Unable to be Loosend) of Nature."**

Dee knew 12 spheres fit perfectly-around-1, as Cardano had noted in 1551, a decade before Dee was writing the Monas.

Bucky and Dee both knew that the cuboctahedron was the star of the show. Dee wanted to shout his discovery out to the world, but didn't want his head cut off by religious intolerants who wouldn't understand this simple geometry of his "atomism." So, he invented the Monas symbol to represent the cuboctahedron!
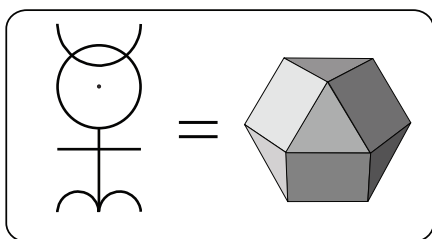
At first glance this seems absurd.
The two things don't look alike at all!
Plus, one of them is a 2-D figure
and the other is a 3-D figure.
Well, that's the whole idea.
The Monas symbol is a really
well-disguised cuboctahedron.

I will show 3 ways they are meant to express the same thing.
I call them:
1) **pumping**
2) **parts**
3) **points**.

Monas symbol = cuboctahedron seen the "Pumping" way.

Dee introduces the concept of two circles (or two spheres) in Theorem 4, cryptically calling them the Sun and the Moon. In Theorem 8, he introduces the idea that the Cross of the Elements can also be seen as an X.
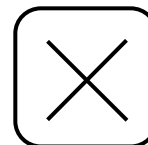
It's hard to conceive of a simple symbol that expresses the union of opposites better than an X. We might depict all of this symbolically like this:

One Circle (Sun)   The Union of Opposites   Another circle (Moon)
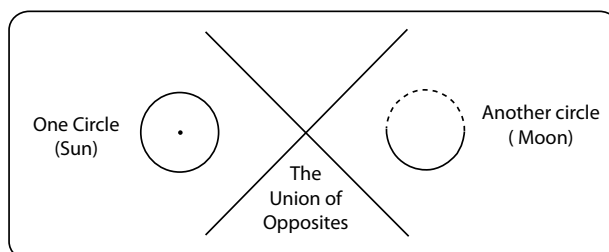
Remember, the "crescentness" of the moon is merely a lighting effect which photographers call "backlighting." If "our eyes" are the camera and the spherical moon is the subject, the light (the sun) is behind the subject. It's not "directly behind" (that would be a totally dark, "new moon", or a Solar eclipse), but very close to being "directly behind." Even during this crescent phase, the moon is still a "round rock." At full moon, there is no question that the moon is round.

As Dee explains in Theorem 4, the Moon yearns to "transform" herself into the Sun. So the Lunar half-circle of the Monas symbol **can justifiably be considered to be a circle**.

Furthermore, the Sun and the Moon appear to be the same size to "our eyes" here on earth (each appearing to be about a half of a degree in diameter).

Let's take it a step further and represent the conjunction of the two circles like this:

(I've shrunk the Moon circle slightly only to make the illustration clearer.)

Sun   Both   Moon
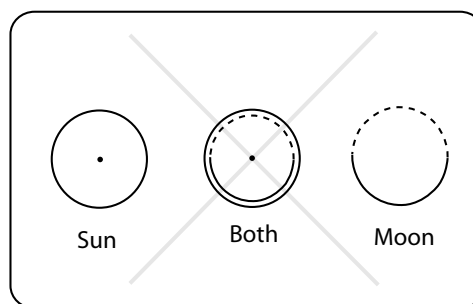
The Union of Opposites

But, the Union of Opposites is really the continuous process of "separatio and conjunctio."

CONJUNCTIO
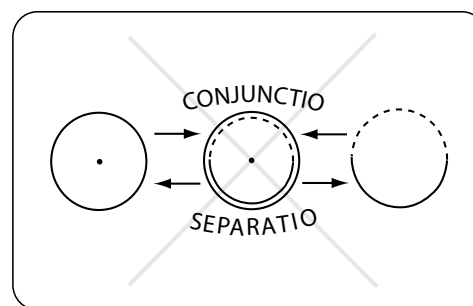
SEPARATIO

The Union of Opposites

We've seen in the "36 Boxes chart," and in many of Dee's key words, that the letters M and T stand for either separatio or conjunctio. In the Monas symbol, these letters are represented by the Cross of the Elements and the Aries symbol (either sign can represent either part of the process).

When these are added to the picture, we have all 4 parts of the Monas symbol. The Monas symbol is an expression of this "pumping action" of conjunctio and separatio, "joining and unjoining" of the union of opposites.

The Union of Opposites

The Union of Opposites
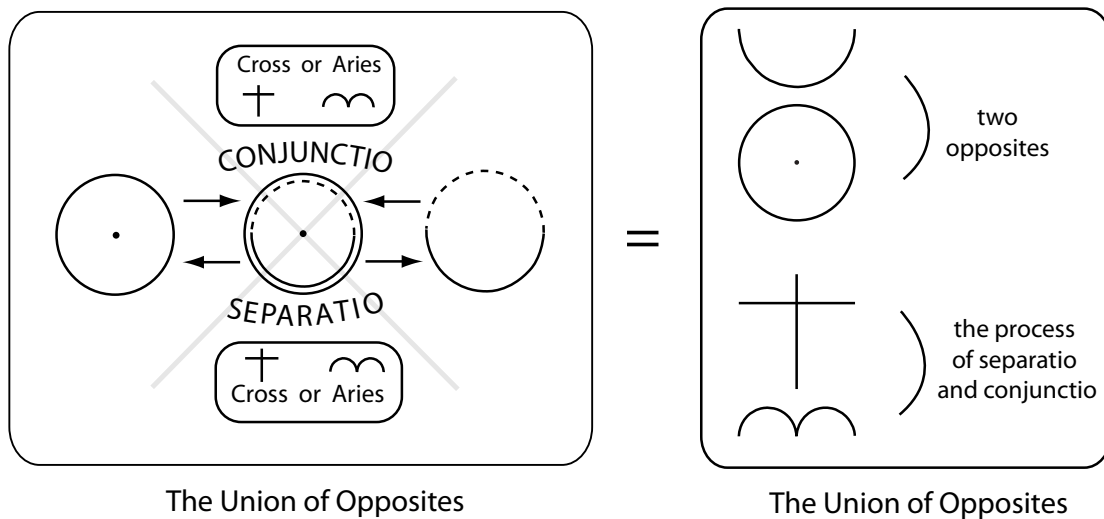
Let's pop from 2-D into 3-D and see these opposing circles as opposing spheres.

The Union of Opposites
using two spheres
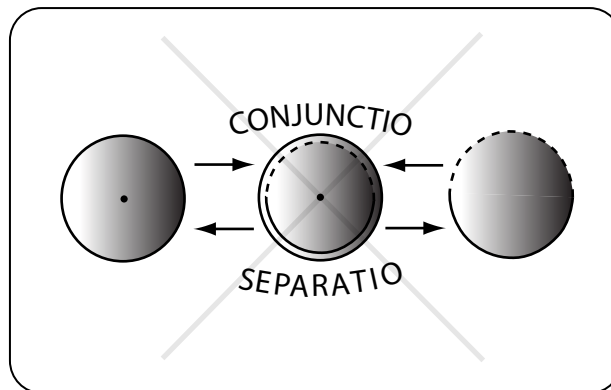
A sphere is a 3-D shape.
Platonic and Archimedean "solids" are 3-D shapes,
whose sides one can actually count.

How many sides are there on a sphere?
Way past a gazillion gazillion.

Bucky's dome
at Epcot
looks spherical...

...but actually,
it's made from
954 triangular panels.

A simpler, 20-sided icosahedron feels somewhat spherical.
A 12 sided dodecahedron feels somewhat less spherical
(unless blown up into a soccer ball).
The 8 sided octahedron and the 6 sided sphere feel considerably less spherical.
But, the tetrahedron feels the least spherical of them all.
In fact, its tips can feel rather sharp.
The chart of "non-sphericity to sphericity" starts at the tetrahedron and ends with the sphere.

From non-sphericity .... to .... sphericity

tetrahedron	cube	octahedron	dodecahedron	icosohedron		sphere
4 sides	6 sides	8 sides	12 sides	20 sides	....	many, many, many, sides
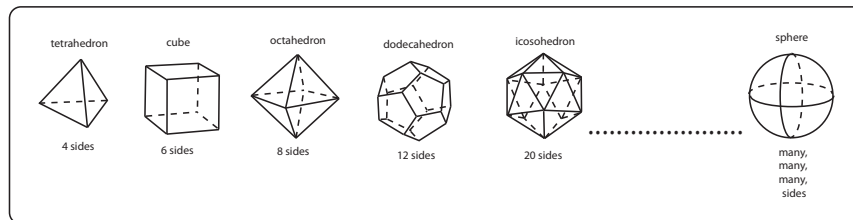
So, let's replace the Sun and Moon spheres
with "**Sun and Moon tetrahedra**."

Now these "opposites" are "united,"
as they share a common point.

It's a Bucky Bowtie!

The "Sun tetrahedron"   The "Moon tetrahedron"

The
Union of
Opposites
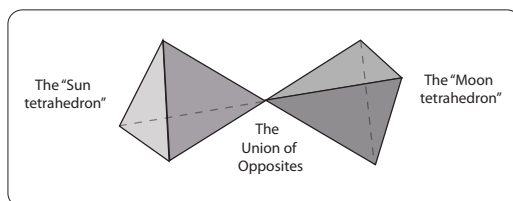
Conjunctio can be seen as the
2 tetrahedra simultaneously shrinking
into that "point of vanishment."

Separatio can be seen as them
expanding outward again
(or even flowing thru the point,
switching sides).

CONJUNCTIO

CONJUNCTIO

CONJUNCTIO

SEPARATIO

SEPARATIO
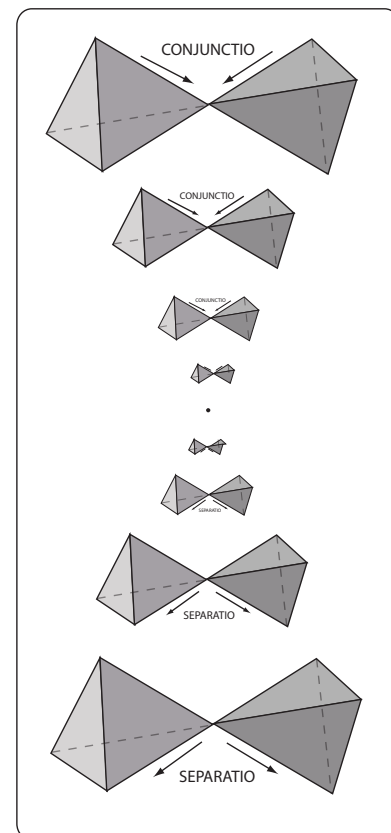
SEPARATIO

Bucky's "pumping" or
"convergence and divergence"
is Dee's
"conjunctio and separatio"

135
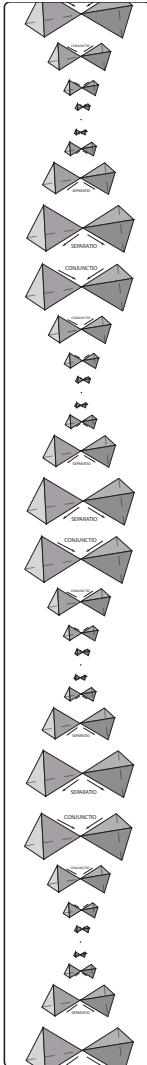
These two tip-to-tip tetrahedra, pumping into the point of vanishment and then back outwards again (then in, out, in, out ...) like some bizarre magic accordion, are very nice, but they aren't a comlete set that makes a nArchimedean solid. There must be 4 pairs of those tip-to-tip tetrahedra (Bucky bowties).

And they all share a common "point of vanishment." Four magic "vanishing and re-appearing" accordions, all playing at the same time. This makes a cuboctahedron.
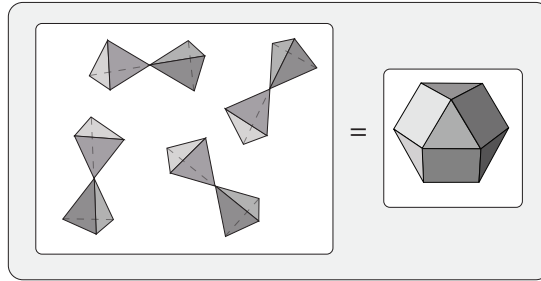
4 pairs of tip-to-tip tetrahedra assemble into a cuboctahedron

Here's a visual summary of the "pumping" reason why the Monas symbol = cuboctahedron.

or
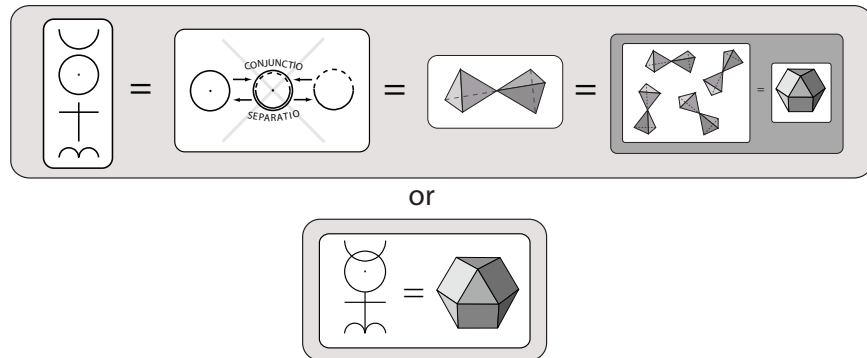
Way 2 :   Monas symbol = cuboctahedron seen the "Parts" way

Bucky saw his vector equilibrium as eight tetradedra, arranged in 4 pairs, all sharing the null ninth centerpoint of vanishment, or the "+4,–4, octave; null nine" arrangement.

Dee depicts the same arrangement in the upper left quadrant of his "Thus the World Was Created" chart. The four squares with the digits 1, 2, 3, 4 and four squares with digits 5, 6, 7, 8 are connected with a giant dotted line X. And, resting above this arrangement is the Horizon number 9, disguised as the "Horizon of Eternity."
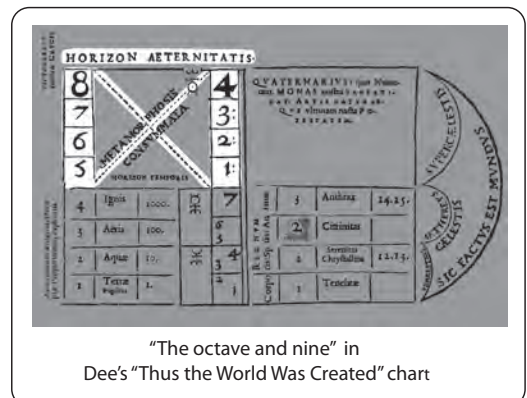
"The octave and nine" in
Dee's "Thus the World Was Created" chart

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But, Dee has cleverly put another set of 4 boxes containing the digits 1, 2, 3, 4 in his chart – the Pythagorean quaternary – the first "quaternary" listed in the "Below" half of the chart.

This Pythagorean quaternary 1, 2, 3, 4, plus the boxes containing the digits 5, 6, 7, 8, comprise the **full height** of the chart. Plus, the Horizon number 9 still sits appropriately above them.

Another way to see "the octave and nine"
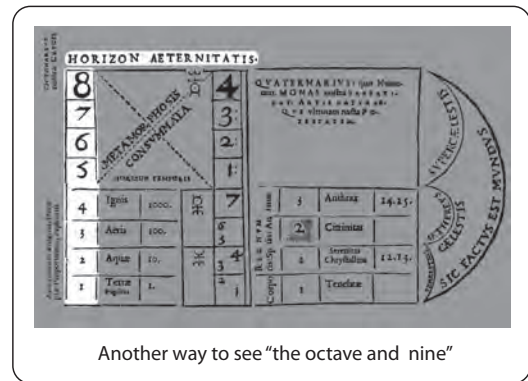
In the geometric construction of the Monas symbol in Theorem 23, Dee shows that the spine of the Monas is made of 10 points. He adds that, alternatively, it can be seen as the "nine equal parts" that are between these 10 points.

So, the Monas symbol and the "Thus the World Was Created" chart can each be seen as expressing this "**nineness**."

The boxes 1, 2, 3, 4, relate to the vertical line of the Cross. The boxes 5, 6, 7, 8 relate to the Sun Circle. And that "null 9" relates to that uppermost space, above the Sun circle
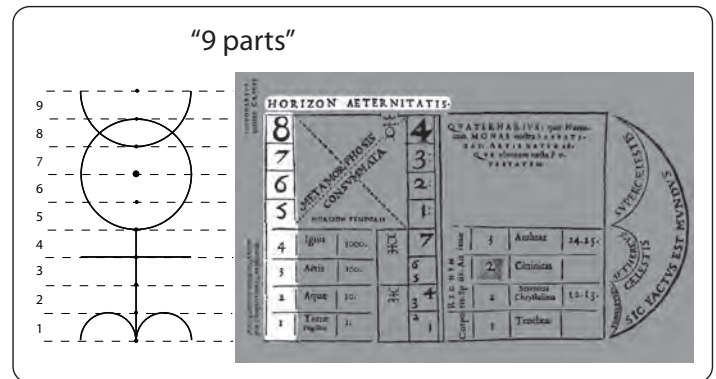
"9 parts"

In this way, the Monas symbol and the "Thus the World Was Created" chart both express 8 tetrahedra, plus the point of vanishment, of the cuboctahedron.
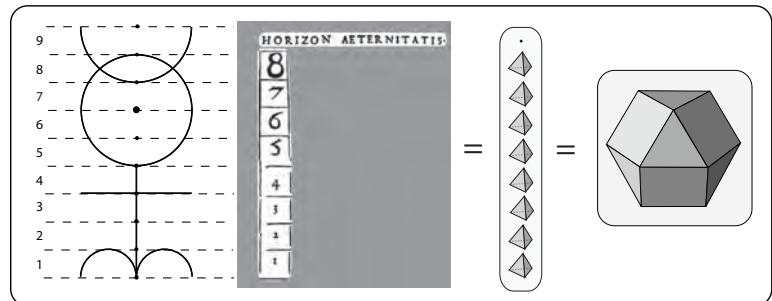
The 9 "parts" of the Monas symbol are the "8 tetrahedra and a central point" from which a cuboctahedron is made.

Question:
How can I presume that the 1, 2, 3, 4, 5, 6, 7, 8, 9 sequence in the "Thus the World Was Created" chart refers to the cuboctahedral arrangement?

Answer:
Because the whole chart "expresses" cuboctahedron!

First, remember that a cuboctahedron has **8 triangular faces** and **6 square faces.**
.
Also recall that Dee has divided the chart into
the "**Below**" half (Terrestrial/Lunary and Solary things)
and an "**Above**" half (Supercelestial things).

In philosophy dating back to the ancients,
a "**square**" (or quarernaries, group of 4 things, like the 4 elements) is associated with the "**Below**"
and a" **triangle**" (or a group of 3 things, like the Holy Trinity) is associated with the "**Above**."

6 Quaternaries in the
"Below" half of the chart

In the "Below" half of the chart,
Dee lists various **quaternaries**.
Let's make an inventory of them:

1) The Pythagorean Quaternary (1, 2, 3, 4)
2) The 4 Elements (Fire, Air, Water, Earth)
3) (1, 10, 100, 1000)
4) (1,2) (3,4) (5,6) (7)
5) The Artificial Quaternary (1, 2, 3, 2)
6) The 4 Alchemical Stages (Black, White, Yellow, Red)

The rest of the information in the "Below" half of the chart is not arranged in quaterna-ries. Thus, in total, there are **six quaternaries**. This is Dee's cryptic way of saying that a cuboc-tahedron has 6 square faces.Thus its a hint that we might find 8 triangular faces hidden in the "Above" part of the chart.
At first glance, it seems like the "Above" half are simply two more quaternaries (1, 2, 3, 4) and (5, 6, 7, 8) and the area between them seems to be a perfect square.

Dee has divided that square with an X,
making 4 triangles.

But, there are only 4 of them, not 8.
And besides,
they are clearly not equilateral triangles.

However, a basic principle of the Art of Perspective is that an equilateral triangle can look like an isoceles triangle, if it not viewed straight on.
For exanple, a bird's-eye-view of an octahedron will look like a square with an X, the same shape that Dee has drawn.

octahedron,
when viewed
from above

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Also, there is more to the "Above" part of the chart: the "upper right" quadrant with that strange "round" sentence.
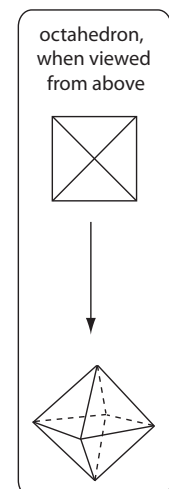
Notice how that sentence has been arranged. Good typesetters try to adjust letter-spacing and word-spacing to avoid hyphenations. But here, the 3 hyphenations make it seem as though the typesetter constrained the type so it would fit in a triangular shape.

Drawing an X across the quadrant shows that it's not perfect – Dee didn't want to be too obvious about it. (The triangular shape is actually easier to see without my translucent grey X.)

These 4 triangles, plus the 4 previously found, make 8 triangles, the number of triangular faces on a cuboctahedron.

"8 triangles" in the "Above" half of the chart

When Dee hand-copied his *Monas* (around 1990, about 26 years after it was pub-lished), he couldn't write small enough to make this triangular shape with the type. But he did center the final syllable of the word POSTATEM.

(The fact that he left the letters TEM at the bottom is a clue that will make more sense after you have read the next few chapters)

Dee's handwritten "Thus the World Was Created" chart (ca. 1590)

In short, the whole "Thus the World Was Created" chart expresses the cuboctahedron.

8 triangles

6 Quaternaries

=

8 triangular faces and 6 square faces of a cuboctahedron
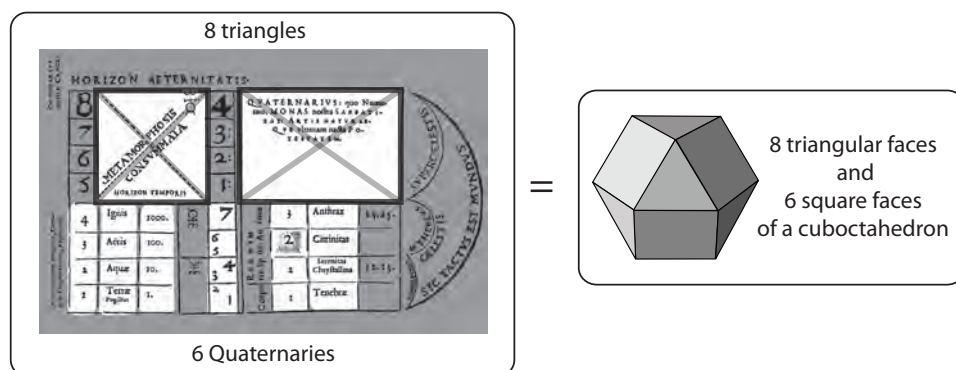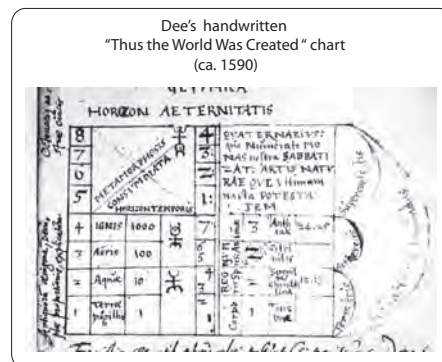
To conclude, the whole chart says "cuboctahedron" (8 triangular + 6 square faces) and the left edge says "cuboctahedron" (octave, horizon number 9), which corresponds to the "9 parts" of the Monas Symbol.
Thus, the Monas symbol expresses the "cuboctahedron" (seen the "parts" way).

Monas symbol = cuboctahedron seen "the Points" way

Instead of the "9 parts," let's focus on the "10 points" of the Monas symbol.
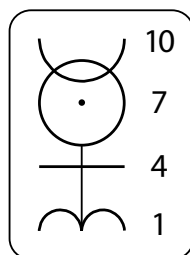
Remember that Paracelsus (and Kelley)
added 3 to the "magical number 4" to get 7.
Then, they added 3 more to make the Denary, 10.
And 10 is a "return to 1."

To depict this,
I have curved the Monas symbol into a circle
so that "1" and "10" share a point.

Monas symbol
curved into a circle
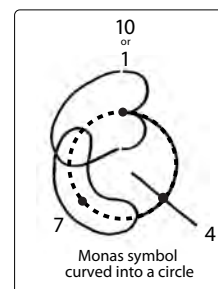
I further contorted it into a triangle,
whose corners were "point 4" (centerpoint of the Cross),
"point 7" (centerpoint of the Sun circle),
and the shared points "1 and 10" (centerpoint of the
Aries symbol and centerpoint the Moon half-circle).

Monas symbol
bent into a triangle
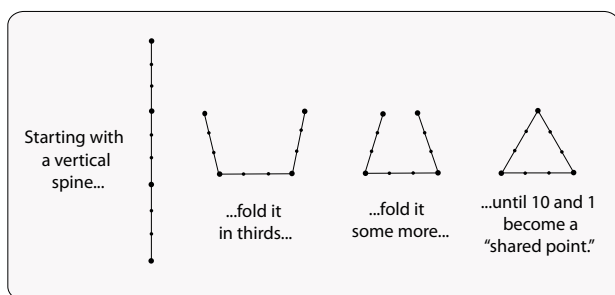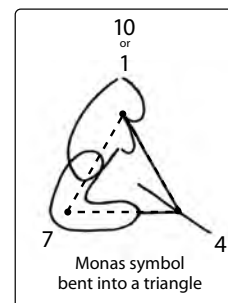
Essentially, we have simply folded
the Monas symbol into thirds
(like folding a letter to fit it in an envelope).
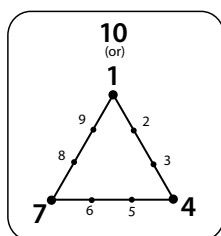
One way to look at this is that
we went from 10 points to 9 points.
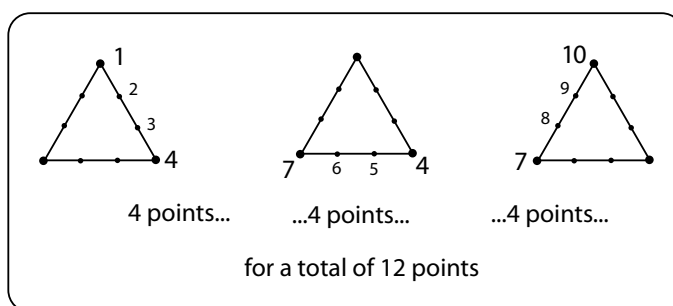
But, there's another way of looking
at it, where there are actually 12 points!

Let me explain:
The right sloping side of the triangle can be seen as 4 points (1, 2, 3, and 4).
The base of the triangle can be seen as 4 points (4, 5, 6, and 7).
And the left sloping side of the triangle can be seen as having 4 points (7, 8, 9, and 10).
This makes a total of 12 points.
Essentially, 4 and 7 have become "shared" points just like the "1 and 10 combination point."

Next, allow me to be a little creative and rearrange each side into a square labeled with its 4 points.

Then, let's let those squares drift back together so they create a central triangle (with the cornerpoints 4, 7, and that 1 and 10 combo.)

Finally, allow me to draw those 3 squares in artistic perspective and connect the outermost points with dotted lines.

Viola!

Suddenly, we have a front view of a cuboctahedron with its distinctive arrangement of triangles and squares!
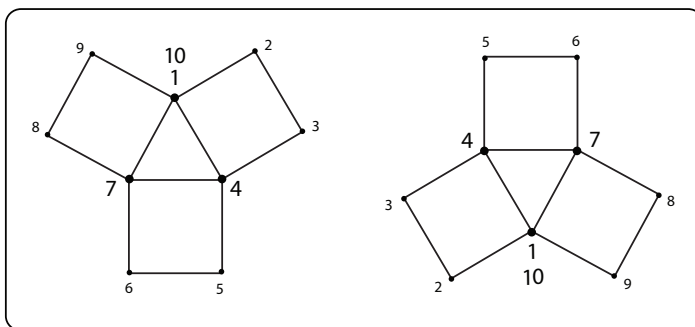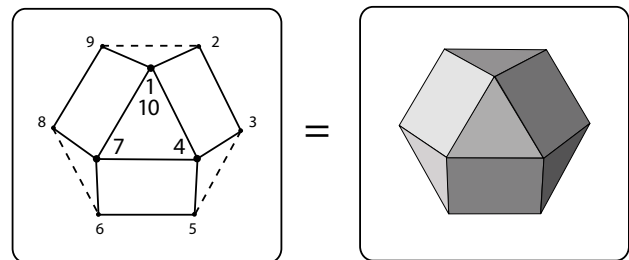
But this only includes the front view.

Let's apply the same procedure to the rear view, (in which the central triangle is pointed downwards).

Front View
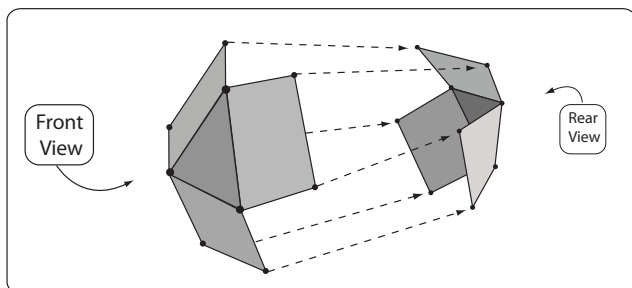
Rear View

The front and rear views
fit together perfectly,
like yin and yang,
like a handshake;
or
like 2 leather pieces
of a baseball.

The 12 vertices of a
cuboctahedron
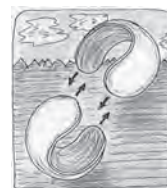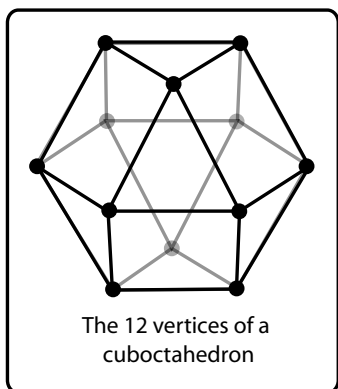
It seems like the 9 points on the front view
and the 9 points on the back view
will total to 18 points.
But, 6 of those points have now become shared points.
As 18 – 6 = 12,
these are the 12 vertices of a cuboctahedron.

The "upright" Monas Symbol needs its opposite, the "inverted" Monas Symbol

This is one reason why Dee's text and illustrations
not only depict the "upright" Monas symbol,
but also the "inverted" Monas symbol.

(mirror shows rear view)

Both symbols
combined...

...make a whole
cuboctahedron

(and)

=
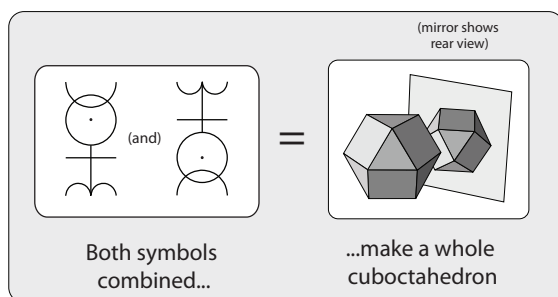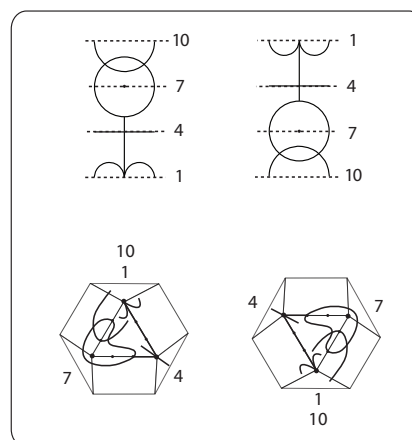
These two versions of the same symbol
are "refections "of each other, just like
the front and rear views of a cuboctahedron.
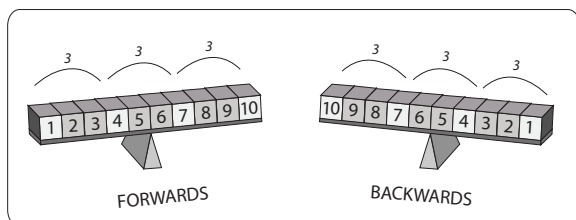
Furthermore, as the spine of the Monas symbol has 10 points, the "Symmetry of the Decad" might be seen as flowing "Backwards" or "Forwards."

More distortion for the purpose of clarification

As if I haven't distorted the distinguished Monas symbol enough, allow me to go a step further to help demonstrate the important link between the Symmetry of the Decad, the Monas symbol, and the cuboctahedron.

Let's start with the two patterns of 3 squares each, shown here with their points enumerated.
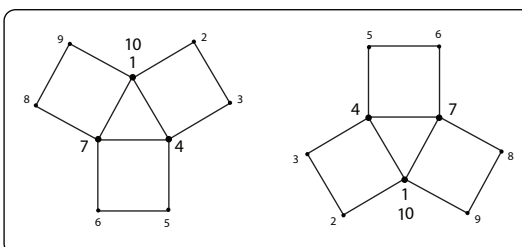
Now, let's contort the Monas symbol so its 10 points correspond with these two arrangements.

Front View Rear View
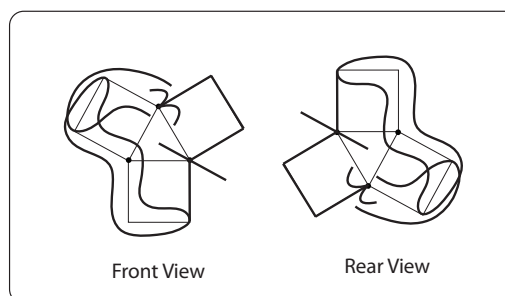
Then morph the arrangements into the front and rear views of a cuboctahedron.

Front View Rear View
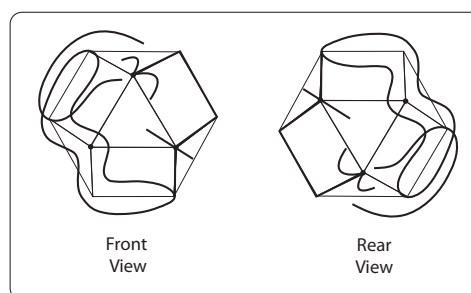
These two severely warped "Modern-Art sculptures" of the Monas symbol actually demonstrate "Monas symbol equals cuboctahedron"

(and) = (mirror shows rear view)

Both symbols combined... ...make a whole cuboctahedron

Front View Rear View

An "upright" and an "inverted" Monas symbol combine to make a whole cuboctahedron

143
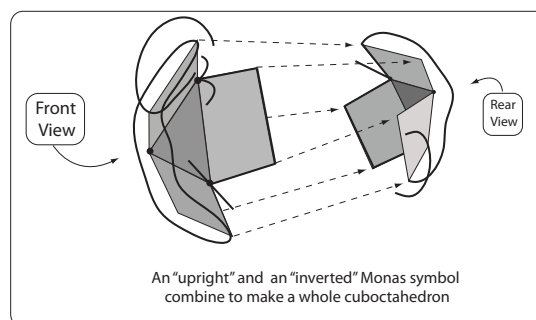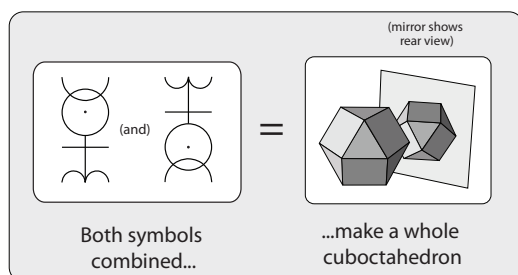
Symmetry of the Decad in Closest Packing of Spheres

Another way to view the
Symmetry of the Decad
in the cuboctahedron is with the
12-around-1
closest packing of spheres
arrangement.

The centerpoints of these 12 spheres
form a cuboctahedron:

Cuboctahedron
of 12 spheres
around 1 central sphere.

The 12 vertices
are the centerpoints
of the 12 spheres.

Cuboctahedron of
12 vertices
and 24 edges
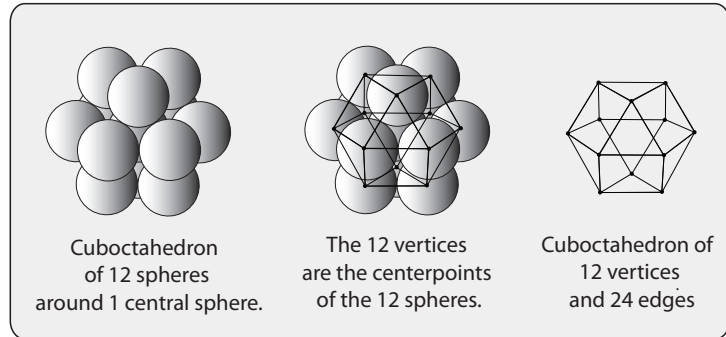
So, we might
enumerate
the spheres
with these "points"
from the spine
of the Monas symbol.
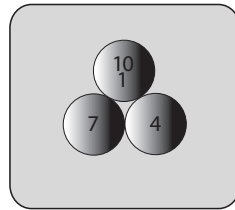
The "rear view"
spheres
would be
enumerated
like this:
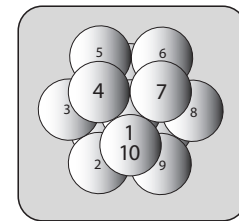
rear view

The 9 front view and 9 rear view
spheres make 18 in total,
but that whole outer perimeter of 6 spheres
is "shared" by both views.
18−6 = 12 spheres

these 6 "outer" spheres are "shared"

front view   rear view

3 spheres   6 spheres
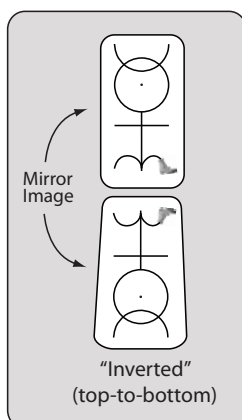(not including
the central sphere)

9 spheres
(of the
front view)
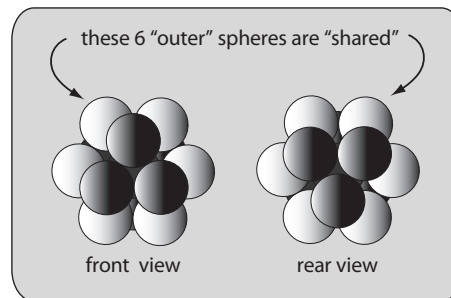
Thus, each "view" the 9 visible spheres
might be seen as a grouping of 3 (in front)
of a grouping of 6 (behind them).

This 3 + 6 = 9 is reminiscent of
Dee's description (in Theorem 21)
of the "3 points" of the Aries Symbol,
*"seen another way are 6,
which added, makes 3 x 3, or 9".*

This 3+6=9
might not seem to be related
to the progression 1, 4, 7, 10,
but remember,
the "Symmetry of the Decad
"is a trio of leaps that are each 3."

(three leaps of three each)

3   3   3

Symmetry of the Decad

(10 points in a line
make 9 spaces
between those points)
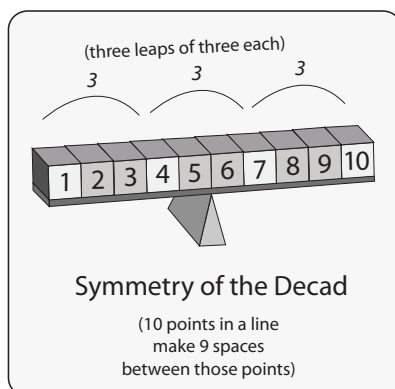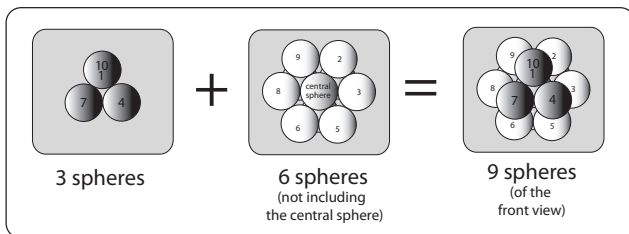
The "inverted and reversed" Monas symbol is the correct terminology
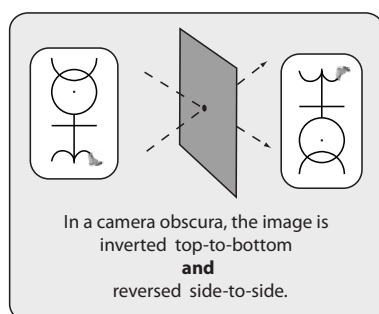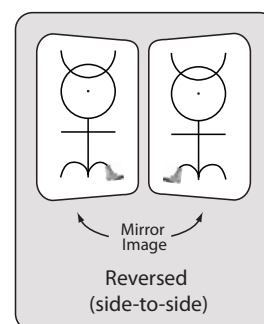

"Inverted"
(top-to-bottom)

Technically speaking,
my term "inverted Monas symbol" isn't correct.
It more properly should be called the
"inverted and reversed Monas symbol."

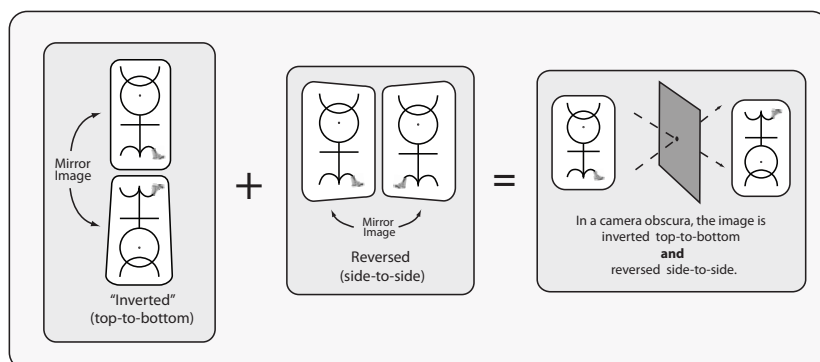To make this all visual, let's put a "**boot**"
on one of the "feet" of Aries.

Here's what simply
"inverted" looks like:

Here's what simply
"reversed" looks like:

Reversed
(side-to-side)

In a camera obscura, the image is
inverted top-to-bottom
and
reversed side-to-side.

Here it is "inverted **and** reversed,"
as in a camera obscura:

In short, "inversion" plus "reversal" equals "camera obscura image.

Because the Monas symbol does not have an asymmetrucal "**boot**,"
this "inversion and reversal" is not visible to the eye.
Still, it's most likely that this is the the way Optics-whiz Dee would have envisioned it.

Despite the importance of this distinction, in this text I will still refer to this "inverted-and-reversed camera-obscura-image Monas symbol" as simply the "**inverted Monas**." This is done only to keep my terminology succinct, but the "reversed" aspect is also being implied.
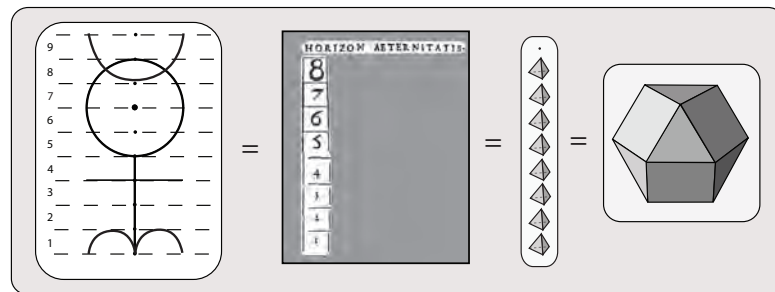
I've done a lot of dissecting and warping here.
Here's a brief visual summary of the 3 ways that the
Monas symbol = cuboctahedron.

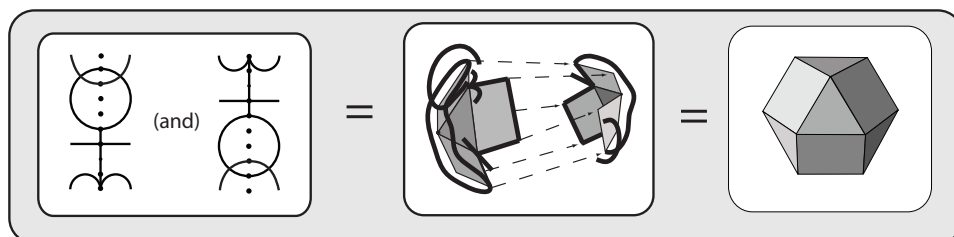### The "pumping" of "separatio and conjunctio" way:
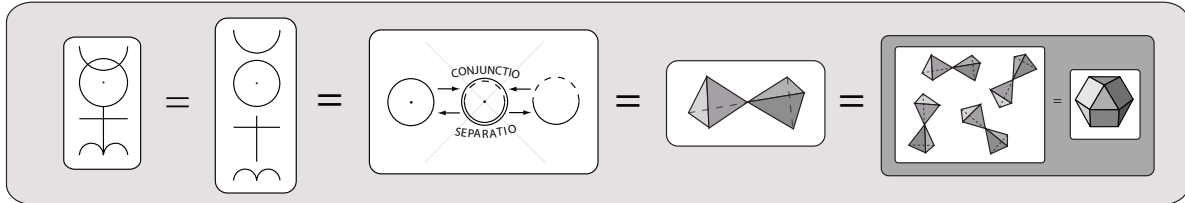
The 4 symbols that make up the in the Monas symbol
express the Union of Opposites,
just like each of the 4 pairs of tip-to-tip tetrahedra
that assemble into a cuboctahedron

The "9 parts" way:

The 9 "parts" of the Monas symbol are the
"8 tetrahedra and a central point"
from which a cuboctahedron is made.

The "10 points" way:

An "upright" and an "inverted" Monas symbol,
seen as a "front view and "rear view"
combine to make a whole cuboctahedron

More suggestions
of the
Cuboctahedron
in the
Monas Hieroglyphica

Once it's understood that
"Monas symbol = cuboctahedron,"
Dee's Title page might be seen as illustrating
some of the various interrelationships between
the cuboctahedron and the Platonic solids.

On the Title page, the central Monas symbol
is surrounded by the 4 Elements.

Let's substitute the cuboctahedron
for the Monas symbol and replace each element
with its associated Platonic solid
**(as per Plato's designations in the Timaeus 53c).**
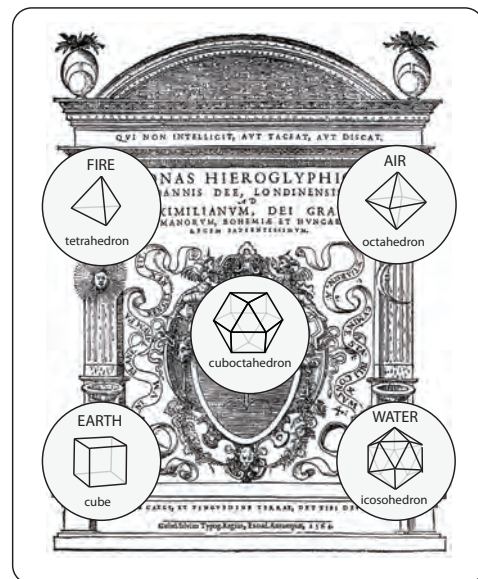
The polyhedra associated with the 4 Elements,
according to Plato's Timaeus.
And in the center of them all,
Dee's Monas symbol
represents a cuboctahedron.

When the Cube and Octahedron meet

One of the most obvious correspondences is that the cuboctahedron is in between the cube and the octahedron!

This "intersection" of the cube and the octahedron was discussed by Flussas in his "brief treatise" appended to the fist English translation of Euclid's Elements.

That special trait of the cuboctahedron, its intermingled triangular and square faces, is inherited from its "parents," the triangular-faced octahedron and the square-faced cube.
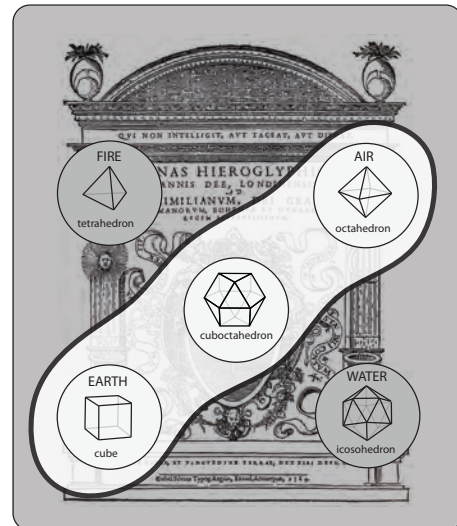

The intersection of a cube
and an octahedron
is a cuboctahedron. .

The Jitterburg

Another grouping includes the cuboctahedron, the icosahedron, the octahedron, and the tetrahedron.

These are the shapes involved in Bucky's jitterbug transformation that starts with a cuboctahedron which has flexible joints. When compressed, it morphs into an icosahedron, then into an octahedron, and then it can be folded into a tetrahedron.

Whether Dee had a cuboctahedron with flexible joints in mind is not known. But it's clear from reading Books 10-16 of the first English *Elements of Euclid* {with its 275 insightful commentaries by Dee and Flussas) that Dee was quite well-versed in the interrelationships between the various Platonic solids. This "jitterbug" relationship is actually quite simplistic compared to many of the interrelationships that Dee and Flussas discuss.
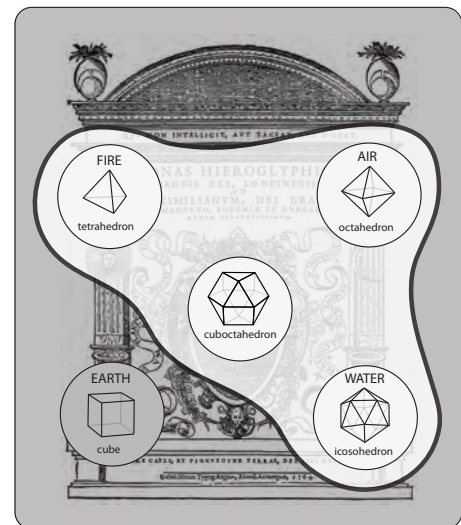

In Bucky's "jitterbug" transformation,
a cuboctahedron morphs into an icosahedron,
then into an octahedron,
then into a tetrahedron.

The octet truss

This next grouping includes the tetrahedron, octahedron, and the cuboctahedron. Recall that a tetrahedron joined with an octahedron is the essence of Bucky's octet truss.
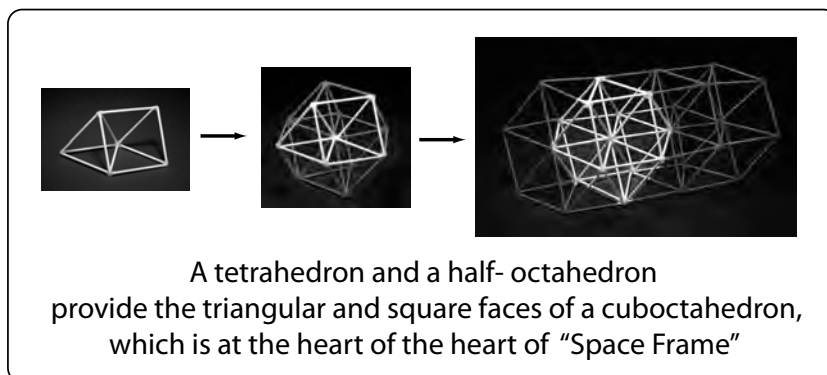
A row of these makes a radio station antenna. A plane of them makes an auditorium ceiling. And many planes makes the "rigid space frame." Radiating from any of the interior points in the space frame is the cuboctahedron.

This is the connection between the tetrahedron, octahedron, and cuboctahedron.

FIRE
tetrahedron

AIR
octahedron

cuboctahedron

EARTH
cube

WATER
icosohedron
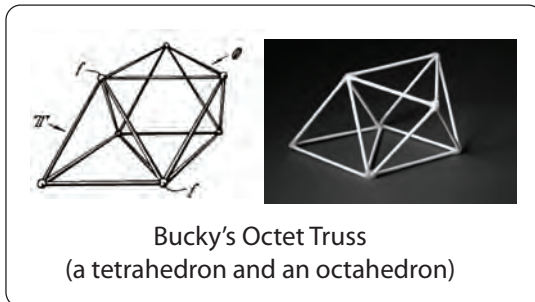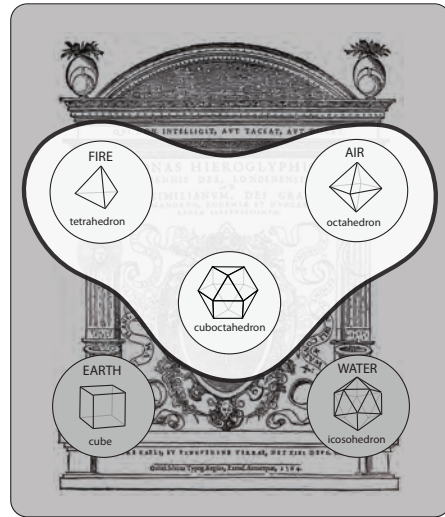
A tetrahedron and an octahedron make
the basic form of the Bucky's "octet truss."
Many of these forms combined make
"space frame,"
inside of which are cuboctahedra.

Bucky's Octet Truss
(a tetrahedron and an octahedron)

A simpler way to see this interconnection
is to start with the arrangement of
the tetrahedron joined to only half of an octahedron.

A tetrahedron and a half- octahedron
provide the triangular and square faces of a cuboctahedron,
which is at the heart of the heart of "Space Frame"

The square shape seen facing the viewer in that half octahedron
produces the **square** faces of the cuboctahedron.
The triangular face of the tetrahedron provides the **triangular** faces of the cuboctahedron.

This is a manifestation of one of Dees' oft repeated maxims:
The Quaternary Rests in the Ternary.
The square faces of the cuboctahedron rest between the triangular faces.

For example, ignoring the "icosahedron" step, the cuboctahedron morphs into an octahedron.
The 8 triangular faces of the cuboctahedron become the 8 triangular faces of the octahedron.
Dee would have known that they are the same 8 triangular faces
(only the square faces have vanished.)

Another example is that Dee would have known that an octahedron
can be divided into two half octahedra.
A half octahedron is a Pyramid of Giza shape made from 4 triangles and a square base.
If these 4 triangular faces are torn apart and re-assembled, a tetrahedron can be formed – and
that square base of the Pyramid of Giza shape has vanished.

To summarize, the Title page can be seen
as expressing these interrelationships between
the cuboctahedron and the Platonic solids.
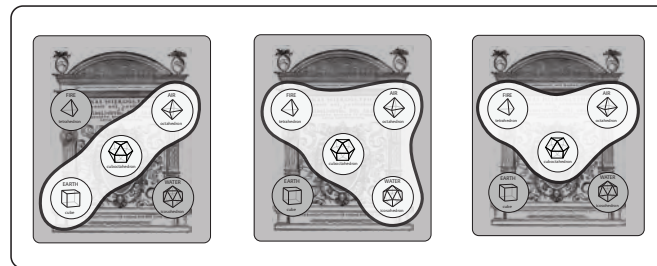
1) cube + octahedron = cuboctahedron.

2) the "jitterbug" of the cuboctahedron...to...icosahedron...to...octahedron...to...tetrahedron

3) tetrahedron + octahedron = octet truss = cuboctahedron

(other interrelationships might also be found, but these are the important ones.)

The cuboctahedron and the
"results" (12, 8, and 24)
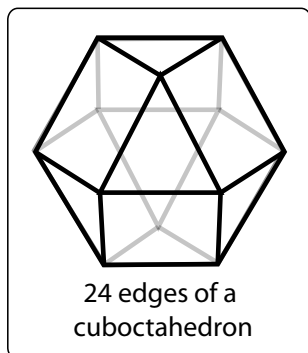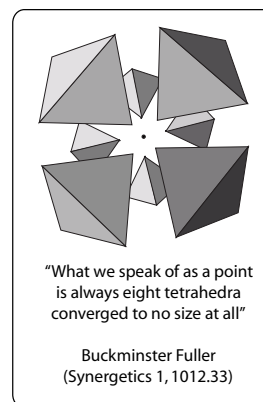### *of Dee's Artificial Quaternary*
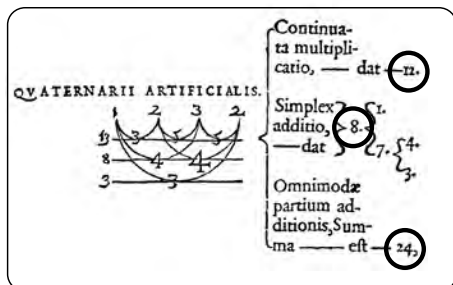
Another cryptic reference to the cuboctahedron is
the 3 results of the Artificial Quaternary of Theorem 23.

We've seen how 12 is a key number in the cuboctahdron.
The cuboctahedron made from vectors will have exactly
12 radiating vectors connecting
the central point to the 12 vertices
.
The cuboctahedron made from spheres will have exactly
12 spheres in the first layer surrounding the central sphere.

We've also seen how **8** is a key number in the cuboctahedron.
Eight tetrahedra sharing a common central point
make a cuboctahedron.

"What we speak of as a point
is always eight tetrahedra
converged to no size at all"

Buckminster Fuller
(Synergetics 1, 1012.33)

24 edges of a
cuboctahedron

And finally, we've seen how the cuboctahedron has **24** edges.
Not only are they all the same length,
but they're the same length as the **12** radiating vectors
(a distinction that no other Platonic or Archimedean solid can claim).

These key numbers, **12, 8, and 24** also appear in the
Artificial Quaternary chart
and the "Thus the World Was Created" chart.

The **12** appears in conjunction with 13,
(like the 12-around-1 = 13 arrangement).
The **24** appears in conjunction with 25,
(like those 24 edges and the 25 great circles of a spherical cuboctahedron).

The number **8** appears in both of the summary charts in the sequence
(1, 2, 3, 4, 5, 6, 7, 8) as if enumerating each of the 8 tetrahedra.

These "cuboctahedraly numbers" are even in title to Dee's book

And if the 12, 8, 24 of the Artificial Quaternary isn't prominent enough,
Dee placed two of these numbers in the most prominent place of all:
in the title Monas Hieroglyphica.
We've observed Dee's fascination with the numerical order of the Latin alphabet.
The word Monas starts with M, the middle or **12th** letter of the 23 letter Latin alphabet.
And the word Hieroglyphica starts with the letter H, the **8th** letter of the Latin alphabet.

Curiously, grammarians over the centuries have not had much respect for the letter H.
David Sacks, in *Letter Perfect*, explains that the
"slightly anemic H" is merely an expelling-of-breath, where all the other letters involve
"expelled breath interacting with elements like the vocal cords, throat, tongue, teeth, or lips."

Around 500 AD, the Roman grammarian Priscian judged H to be "not a true letter."
In 1529, Geoffroy Tory in *Champ Fleury* wrote, "the aspirate is not a letter;
none the less it is by poetic license given place as a letter."

Sacks notes that many of our words derived from Medieval French
still have a "silent h", like "heir, honest, honor, hour. . . ."
(Sacks, pp. 157,161,168)

The ancient Greek word for hieroglyphic is "**ieroglyphikos**"
(from "ieros," sacred + "glyphe," carving.)
It was the Romans who added the h, making "**hieroglyphicus**."
Thus, in a cryptic way, Dees' title word "Hieroglyphica"
might also be seen as "Ieroglyphica,"
which means it begins with the 9th letter.

As it might start with **either H or I**, this word cryptically seems to be referring
to the eight tetrahedra gathered around the central ninth thing,
their common point, which is a geometric expression of the
"octave, null 9" rhythm that Bucky perceived in the realm of numbers.

Bucky's Synergetics in Dee's Monas Hieroglyphica
(Taking a brief side trip to Utopia)

**Bucky and Dee were
on the same wavelength**

JOHN
DEE

You're probably wondering how I can claim that Dee knew the principles of *Synergetics*. Admittedly, there is a 400 year gap, during which time incredible advances were made in math, geometry, and all the sciences. Then suddenly Bucky comes up with the same thing?

Because Dee wrote cryptically, his knowledge was lost – it hit a dead end. Sure, some of his fellow Elizabethans (and Jacobeans) and alchemists on the Continent knew what the *Monas* meant, but their knowledge of it seems to have to died somewhere along the line of history.

BUCKMINSTER
FULLER

There are several good reasons why Dee and Bucky both were able to comprehend this knowledge.

First, they were both actively looking for it. Dee searched for the Laws to Nature. Bucky searched for "systems operating of Universe. They both worked fervently in their pursuits.

Second, they thought alike. They had creative minds that saw the world in terms of opposites. Dee got it from alchemical thinking of "separatio and conjunctio," and "coincidentia oppositirum," the union of opposites. Bucky intuited "inside-outing" and "convergence/divergence," "gravity/radiation," simply from his own creative observations of the world.

Third, each of them had a broad knowledge base. Dee, with his 4000 book library and his associations with the leading mathematicians on the continent, has been called a Renaissance "Magus" or polymath (poly: much + manthanein: to learn). Bucky called himself a "comprehensivist." As Bucky puts it in Synergetics:

Closest Packing of Spheres
Nature's Coordination

**About 1917, I decided that nature did not have
separate, independently operating departments
of physics, chemistry, biology, mathematics, ethics, etc.**

**Nature did not call a department heads' meeting
when I threw a green apple into the pond,
with the department heads having to make a decision
about how to handle this biological
encounter with chemistry's water
and the unauthorized use of the physics department's waves.**

**I decided that it didn't require a Ph.D. to discern
that nature probably had only one department
and only one coordinate, omnirational, mensuration system**.

(Fuller, *Synergetics I*, 401.011, p. 108)

And the final reason is that they were both expert geometricians. They both felt felt the clues to what they were searching for could be found in geometric shapes and arithmetic numbers.

Some clues that Bucky and Dee discovered the same thing

Here are the clues in the Monas Hieroglyphica that led me
to understand that it is the same as Synergetics.

More about 252

As explained previously, Dee's "252" was the clue that led me to Bucky. Dee's Magisterial Number 252 is the number of spheres in the 5th layer of closest packing of spheres.

In Theorem 17, Dee derives 252 from various numbers related to the Cross of the Elements in an unusual way.

$$20 + 200 + 10 + 21 + 1 = 252$$

He writes that there are "**two other logical ways**" to derive 252. These are mathematical ways, as we shall see later, but he also adds:

**". . . various artificial productions
of this Magisterial Number are also
worthy of the Consideration of Philosophers."**

The term "artificial production" means "something crafted with skill." This might be a skillful mathematical caculation, or a 2-D geometric drawing, or a 3-D geometric model – like a model of five layers of closest packing of spheres.

Just because Elizabethans didn't have ping-pong balls, lollipop sticks, and hot glue doesn't mean they couldn't have made models involving spheres and sticks.

Even the Egyptians and the Greeks knew how to make glue. During the Renaissance, glue made from animal parts, beeswax, resin, or tar was used by furniture makers and shipbuilders.

As for small spheres, people have been playing with marbles for over 3000 years. They weren't necessarily the perfectly smooth glass marbles we have today, but small spherical clay balls have been found in Egyptian and Aztec excavations.

It would only take a few hours to roll out the 571 small spheres required to construct a 5-layer "closest packing of spheres" configuration. (Clay spheres might even have been glazed. Pop them in the bread oven for an hour and you're ready to start gluing.)

Bowles, balls, and bullets

Dee makes a revealing remark in his "Addition" to the last proposition in the 12th chapter book of the English version of Euclid. He is discussing the "spherical solidity" or the volumes of the "earthly sphere" and "heavenly spheres."

"... I trust I have sufficiently fraughted [supplied] your imagination for your honest and profitable study herein, and also giving you ready matter with which to fill the mouths of malicious, ignorant, and arrogant despisers of the most excellent mathematical discourses and inventions.

Seeing as well, the heavenly spheres and stars, their spherical solidity, with they are convex spherical superficies [surfaces], to the earth at all times respecting, and their distances from earth, and also the whole earthly sphere and globe itself, and infinite other cases concerning spheres or globes, may hereby with as much ease and certainty be determined of, as the quantity of any **bowle, ball, or bullet** which we may grip in our hands (reason and experience, being our witness).

And without these aids, such things of importance will certainly never be able of us, certainly to be known or attained unto.

Here end M. John Dee, his additions upon
the last proposition of the twelfth book."
(Dee in *Euclid*, p.389 and 389 verso; emphasis mine)

[Bowle does not mean a hemispherical soup bowl.
It means a round ball used for bowling, a popular sport in Elizabethan times.
The bowle was rolled (or sometimes thrown) at the 10 pins]

***Another clue: Much like the cuboctahedron,
the Monas symbol cryptically symbolizes
the continuous process of Separatio and Conjunctio,***

Dee loved **books**. He had the largest library in Elizabethan England.

Dee loved **words**. He was fluent in Hebrew, Greek, Latin, most of the Romance languages, and, of course, English. Just reading the names of the Mathematical Arte in his Preface to Euclid, you can tell how much he loved etymology and indeed even coining new words.

Dee loved **letters and their shapes**. One example of this in the Monas is his crafting the word LVX from right-angled letters that form crosses in various ways.

Dee loved **codes**. He even professes his use of Gematria, Notarikon, and Temurah in the Monas. He loved the idea that letters can represent some other thing or concept. The triangular Greek letter Delta was his self-portrait.

To investigate the symbolic meaning of the Monas symbol, let's start with the
words and letters in the unusual **"36 Boxes"** chart of Theorem 22.

At first glance,
it seems like a hodge-podge
of various theological ideas.
Further study shows that
they are various "epochs,"
each divided into a
beginning, middle, and an end.

In fact, one of the trios is
Principium, Medium, Finis,
or "Beginning, Middle, and End."

Next to it is a reorganization of the parts of the Monas symbol.

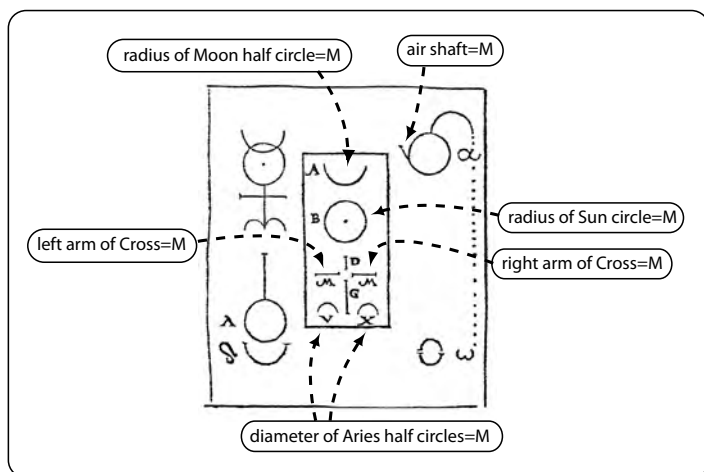The Sun and Moon combine to make a small Greek **alpha** – the first Greek letter.

The Aries symbol has been inverted to resemble an **omega** – the last Greek letter.

In the **middle** is Dee's Cross of the Elements. It seems as though it should represent the middle letter of the Greek Alphabet, but neither Chi (X) nor Tau (T) hold that position. In fact, there is no middle letter in the 24 letter Greek alphabet, the center point falls in between Mu (M) and Nu (N).

But the Latin alphabet, which has only 23 letters, there is a letter "midway" between A and Z; namely, the **letter M**. We know this kind of stuff is of interest to Dee as he points out in Theorem 16 that the "**letter L is midway between A and X.**"

The Letter M

Dee refers to the letter M several times in Theorem 22. In what he calls the "Anatomy of the Cross" in the "Secret Vessels of the Holy Art" diagram, the left and right arms of the cross are labeled M and M. [This was long before Hershey's made their melt-in-your-mouth chocolate candies or rap singer Marshal Mathers adopted the moniker Eminem.]

radius of Moon half circle=M   air shaft=M
left arm of Cross=M   radius of Sun circle=M
right arm of Cross=M
diameter of Aries half circles=M

Dee points out how he has crafted a circular distilling vessel out of the small Greek letter alpha (α).

He has combined the Sun circle and Moon half circle, both of which he says "have the same RADIUS, namely "M".

Later in the Theorem, he notes that the "straight line appearing in Alpha" (or the air shaft in the distilling vessel) is "homologous to the part marked by the letter M in the most recent Anatomy of the Cross."

Dee also says that the words in the 36 Boxes chart give "**Conclusive Proof**" that this Doctrine is "**not Mythical, but Mystical.**" Here are two more M's, each of which Dee capitalized in the original Greek, which he intermingles with Latin in the same sentence (non Mythaxon . . . sed Mystixon).

The fact that Dee emphasizes the letter M so much and that it's the "Medium" letter between alpha and omega, it seems as though he wants us to see it as an Latin alphabet "balance point" or "centerpoint" or "**m**edium."

ABCDEFGHIKLMNOPQRSTVXYZ
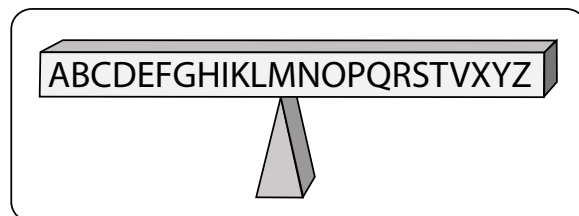
1	A	Z	23
2	B	Y	22
3	C	X	21
4	D	V	20
5	E	T	19
6	F	S	18
7	G	R	17
8	H	Q	16
9	I	P	15
10	K	O	14
11	L	N	13
	M		
	12		

Another way to see it is the alphabet folded-up on itself with the middle letter isolated from the two sides. As Dee was also quite interested in their associated numbers.

In Theorem 16, he explains that,
"V" is Latin letter 20 and
"X" is Latin letter 21.

At the end of Theorem 24, he notes that
"D" as Latin letter 3.

It's pretty obvious that he saw "M" as Latin letter 12.

If this 12 refers to some kind of measurement (**for example, feet**), the diameter of the Sun circle or the width of the Monas symbol is 24 feet. Thus, the height of the whole Monas symbol would be 4.5 times 12, or 54 feet.

We know these numbers 12 and 24 are important to Dee, as in Theorem 11 he says the "24 Hours of Time" of the first of Aries is divided exactly into 12 hours of daylight and 12 hours of darkness, denoting "our most secret proportions."

If radius of the Sun circle
is 12 units,
the height of the whole symbol
is 54 units
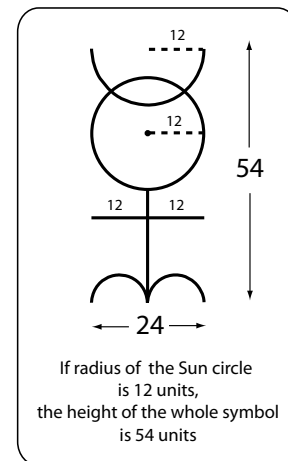
I got the sense that Dee was trying to express the symmetry of the Latin alphabet using the Greek "alpha and omega" to hint at the "A and Z" of the Latin alphabet, the "medium" of which is M.

All this "M-ness" alerted me to the fact that there were an abundance of 'M-words' in the 36 Boxes chart. Surrounding the last trio of Principium, Medium and Finis are the words **Matrimonium** (Marriage), **Martyriú** (Martyrdom), and **Manifestimus** (Manifestation). Around the chart there were more of them – **Mortalis**, **Masculus**, **Mortificins,** as well as words containing M's, like **ADAM IMMORTALIS**, **Elementa**, **Adumbratus**, **Holocaustum**, **Regum**, and **Transformatio**.

Some boxes appeared to be hiding M's just out of plain view. For example, Dee writes the word "Triumphant" as **Triúphus** with an accent over the "u" indicating that the letter which follows it has been omitted. Dee uses this writing technique frequently, but it's curious that it's an **M** that is missing here.

The box which reads Natus in Stabulo (Born in a Stable) contains the noun stabilo, which is a form of the word "**stabulum**" (stable), which has an M.

Regarding the box "**Creatio Hyles**," the most common synonym for Hyle is "**Matter.**"

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The two twin boxes with the word **Crux** in them really stand out as unusual repetition. If they are seen as two side by side crosses, one might imagine them creating an **M** (as well as an inverted **M**, (the letter W wasn't popularized until around 1600).
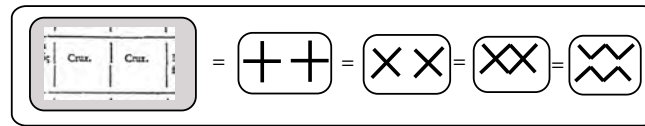
Similarly, just below one of the Crux Boxes is the word **Vivificans**. Not many words can claim to have 2 V's in them. Dee has told us in Theorem 26 that two V's can be made into an X if one of the V's is inverted. If both V's are inverted, we might see them as making an M.
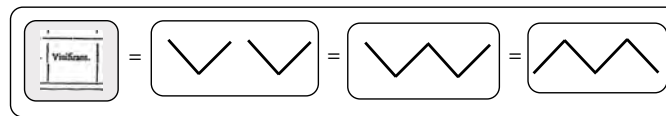
The letter T

What's puzzling is that even though the word Medium starts with an M,
it is associated in this chart with the Cross of the Elements,
which Dee tells us can be seen as an X.
But, sandwiched between a symbol crafted to look like the letter alpha,
and a symbol crafted to look like an omega,
that Cross of the Elements most closely resembles
a lowercase Latin letter "t."

In the English language, the letter T
is the "most commonly used" consonant.
Dee never mentions that the
Cross looks like a lowercase "t,"
but he does encourage the reader to see the
Cross creatively, as letters
(an X, two L's, two V's)
and even as numbers (in various theorems)
as 1, 2, 3, 4, 5, 7, 8, 10, 20, 21, and even 252.

A quick glance at the chart shows that almost every Box
(not including the Monas symbol parts or the Boxes labeled Principium, Medium, and Finis)
include the letter "t" (or T).
The 4 boxes that don't contain a "t" (or T) do have an X (or 2 V's).

Dee placed the 36 Boxes chart within the text of a Corollary
(*Porisma* in Dee's Greek) to the main part of Theorem 22.
This corollary starts out with the Greek words
"**Tes ieras Texnes**" ("The Holy Art").
The Greek letter Tau is prominently capitalized in "**Tes**"
and "**Texnes**," even though "ieras" (Holy) is not capitalized.
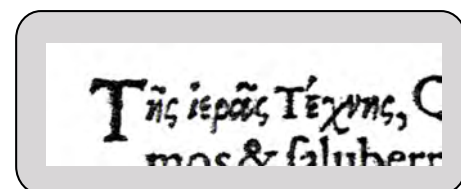
Many of those words which contain the letter **M** also contain the letter **T**.

Matri**m**oniu**m**

Matryiu

Manifes**tim**us

Ele**m**en**t**a

Ele**m**en**t**aris

Mor**t**alis

ADA**M** I**MM**O**RT**ALIS

Mor**t**ificans

Adu**m**bra**t**as

Holocaus**tum**

Tri(**m**)phus

Transfor**m**a**t**io

(Certainly, there are other frequently appearing letters in the 36 Boxes Chart like the vowels AEIOU, and the letter S.) Dee does use these letters symbolically as can be seen in Aphorism 18 of this Propadeumata Aphoristica where he highlights A, O, and S. (But that's a slightly different code than he's using in this chart, which I'll explain later.)

A Man of all Hours

A confirming clue that he's hinting at the letters T and M in the 36 Boxes Chart
can be found in the sentence that precedes the Corollary:

"ω, autem, Omnium est HORARVM Homo"
or
"omega, however, is a Man of All HOURS"

This sentence stands out because it is centered in the page (not right and left justified like the rest of the text) and because the word HORARVM is typeset in such large letters.

At first, this seems like Dee is simply saying that omega, the 24th Greek letter, is like the "24 HOURS of a day."

The Latin word HORA means "an hour," but poetically it also means a "time" or "season." Thus, the sentence could be seen as **"ω, however, is a Man for All SEASONS"**

Dee has dropped a literary clue that any Elizabethan and most learned Europeans would have understood – and curiously anyone who lived in the 1960's would also get. The Best Movie of the Year, in 1967, winning 6 Oscars was ***"A Man for all Seasons."*** This movie is story of Thomas More, the English humanist (1477–1535) who was beheaded for refusing to accept Henry VIII as head of the Church of England.

The quote actually comes from the author's preface to the 1509 *The Praise of Folly* written by the "greatest classical scholar of the northern Humanist Renaissance," Desiderius Erasmus (1466–1536).

Erasmas' title in Greek is *Morias Enkomion*, which can also be read as "*In Praise of More*." The book is a satirical examination of the Roman Catholic Church and it became an influential catalyst of the Protestant Reformation.

Here is the beginning of the preface, dedicated to "my dear More," as translated by Robert M. Adams in 1989:

Sir Thomas More

"Recently when I was on my way from Italy to England, instead of wasting all the time I had to spend on horseback in idle chatter and empty gossip, I tried occasionally to think over some of the things we have studied together, and to call to mind the conversation of my most learned and agreeable friends from whom I was then separated.

Among those friends, you, my dear More, were the first whose name occurred to me, since I find just as much pleasure in thinking of you when we are apart as I do in your company when we are together. And, upon my soul, nothing in life has ever brought me more pleasure than your friendship.

Well, since I felt I must be doing something, and the circumstances were hardly proper for serious study, I thought I might occupy myself with the praise of folly. What put such a notion inn my mind? you may ask.

My first hint came from your family name of More, which is just as close to Moria, the Greek word for folly, as you are remoter from the thing itself. In fact, everyone agrees that you're as far removed from it as possible. Besides, I had a suspicion that this joke would be agreeable to you because you particularly enjoy jests of this sort – that is, if I don't flatter myself, jests seasoned with a touch of learning and a dash of wit.

For that matter, you enjoy playing the role of Democritus in all the common business of life. Though as a result of your searching and original mind you're bound to hold opinions very different from those of common men, **yet by virtue of your warm and sincere manner you can get along with all sorts of people at any time of day**, and actually enjoy doing so.

Will you then accept this little declamationlet of mine as the keepsake of a friend, and take it under your protection? For now that it is dedicated to you, it is properly yours, not mine. I don't doubt that there will be busybodies to condemn the book, some saying that it's composed of trifles too silly to befit a theologian's dignity, others declaring that it's too sharp of tooth to accord with the modest behavior of a Christian – they will thunder out comparisons with the Old Comedy and satires of Lucian, they will say I snap and slash at everyone like a mad dog."

The sentence which I've highlighted, in the original Latin, reads:

**Ita pro incredibili morum suavitate facilitateque cum
omnibus omnium horarum hominem agree et potes et gaudes.**

C. H. Miller (in 1979) translates this sentence this way:

**"the incredible sweetness and gentleness of your character
makes you able and willing to be a man for all seasons with all men."**

The "man for all seasons" interpretation goes way back to More's friend Robert Whittington, who praises More while borrowing Erasmus' expression:

"And as tyme requireth
a man of merveylous myrth and pastimes,
& sometyme of as sad gravyte
as who say a man for all seasons"

(White, B., *Early English Text Society*, Vol 64, pp. 35-7)

So, Dees' "ω, autem, Omnium est HORARVM Homo" is a very obvious literary reference to **Thomas More**, whose initials are **T** and **M**.

A confirming clue can be found in the **only** box in the "36 Boxes chart" with capitalized letters:
ADAM IMMORTALIS,
which is cleverly hyphenated to emphasize
the syllable "MOR-" followed by a "T."
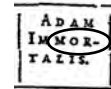
The suggestion of "More," as in Thomas More

More about Utopia

Thomas More is famous for writing *Utopia* (1516), which describes a fictional island in the Atlantic Ocean. It has a society and government that seems to work perfectly. The main fictional character is named Raphael Hythloday, a sea captain who, as More puts it:

. . .originally joined Amerigo Vespucci,
and was his onstant companion in the first three of four voyages
which everyone is now reading about;
but on the last voyage he did not come back with him.

He sought and practically wrested from Amerigo
permission to be one left behind in a fort at the farthest point of the last voyage.
And so he was left behind in accordance with his outlook,
since he was more concerned about his travels than his tomb.

Indeed he often used to say,
'Whoever does not have an urn has the sky to cover him' and
'from everywhere it is the same distance to heaven'

(More, in Miller, p. 12)

More's fictional work borrows from reality. In 1507, two Latin narratives were published about the Voyages of Amerigo Vespucci, from whom we get the name America. Amerigo was Italian, but he was commissioned by the King of Portugal. One of the narratives mentions 24 mariners who were left behind on Cape Frio, on the southeastern coast of Brazil.

(Miller, footnote 36, p. 144)

Do you recognize the final sentence: "from everywhere it is the same distance to heaven." These are, in essence, the final words of **Sir Humphrey Gilbert** just before his ship capsized in a wild tempest off the Azores in 1583:

"**We are as near to heaven by sea as by land**."
Edward Hayes describes Gilbert as holding a book in his hands
that many historians believe was More's *Utopia*.
Gilbert, Dee, Peckham and all had envisioned a Utopia
in the New World – a well-planned Utopia
which died when Sir Humphrey's ship went down.

It's worth re-counting Haye's narrative of that climactic moment:

Monday, the 9. of September, the afternoon,
the frigate was near cast away, oppressed by waves,
yet at that time recovered; and giving forth signs of joy,
the General, sitting abaft [on the stern] with a book in his hand, cried out to us in the Hind,
so oft as we did approach within hearing,
We are as near to heaven by sea as by land!
Reiterating the same speech,
well beseeming a soldier, resolute in Jesus Christ,
as I can testify he was.

(Edward Hayes, Sir Humphrey Gilbert's Voyage to Newfoundland, 1583, Modern History Sourcebook.)
(emphasis mine)

(Incidentally, *Utopia* had been translated from Latin into English in 1551 by Ralph Robinson.)

More about Raphael Hythloday's saga

"However, after the departure of Vespucci, he traveled through many lands with five companions from the fort, and finally, by an extraordinary stroke of luck he was transported to Ceylon and from there he reached Calicut [not Calcutta, but on the coast of India], where he opportunely found some Portuguese ships and at last, beyond all expectation, he got home again."

(More, in Miller, p. 12)

During this journey, Hythloday visited an island he calls "Utopia," a pun on the Greek words "ou-topos,"which means "no place" and "eu-topos" meaning a "good place." We get our word "utopia" directly from the title of More's book.

Hythloday described the land's features, culture, and politics, contrasting this "ideal society" with contemporary Europe:

On Utopia there was no private ownership.
People simply requested what they needed.
All doors were unlocked.
Every 10 years, the citizens rotated houses.
All those who live in cities worked in the country (for two year stints), to assist with farming.
There was a six-hour workday, free hospitals, free food, and freedom of religion for all.
There was no hunting, gambling, or astrology in Utopia.
People used no make-up and they all wore the same types of simple, homemade clothes.

Incidentally, More wasn't the first to describe an ideal society. Plato, in The Republic, was one of the first. Other authors of this genre who were influenced by More's ideas were Tommaso Campanella, (*The City of the Sun)*; Johannes Valentinus Andrae, (*Description of the Republic of Christianopolis)*; Francis Bacon, (*New Atlantis)*; Voltaire, (*Candide)*; and George Orwell, (*1984)*. (Wikipedia, Utopia, p.3)

In Book 2, Raphael Hythloday describes the island of Utopia as having a tower at the north of a large bay.

The illustration which More included in his book is quite similar to the Title page of John Dee's General and Rare Memorials pertaining to the Perfect Art of Navigation, published in 1577, a few years before the planned settlement at the John Dee River and Port.

Incidentally, Raphael Hythloday is a tongue-in-cheek name itself. Raphael means "God's healer" and Hythloday means "peddler of nonsense."

(Miller, intro, p. 10)

Sir Thomas More's
Utopia
(1516)
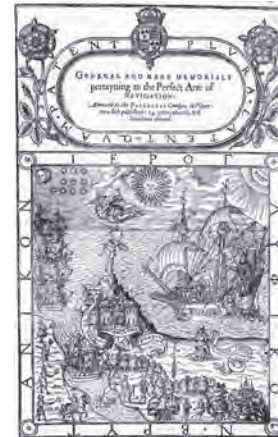
John Dee's
General and rare memorials
pertaining to the Art of Navigation
(1577)

Anagram for "Sir Thomas More" and also "Raphael Hythloday"

(The missing "ph" can be found in "Triumphus")

Another clue is that these two neighboring boxes ("**ADAM IMMORTALIS**" and "**Creatio Hyles**") appear to be an anagram for
"**Sir Thomas More**" and "**Raphael Hythloday**."

They contain all the letters (if some are used several times) except the "ph" in "Raphael," which can be found in the "Gloriae Triúphus" box, which preceeeds the Creatio Hyle box.
(Incidentally, "Gloriae Triúphus" contains all the letters of "Utopia.")

Yet another example of Dee's word-play is the word "oeconomia" in the Box "Elementaris oeconomium." Dee seems to be an punning with the word "encomium" in the title of Erasmus' book *Moriae encomium, (In Praise of Folly)*.
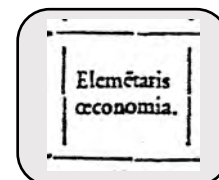
The Utopian Alphabet

When Utopia was first published, people "took the story literally: they assumed that Utopia was a real place and that Raphael Hythloday had visited it."

This was partly caused "Mores' straight-faced narrative technique, but More and his friends Erasmus and Peter Giles published letters that made the whole tale "appear" authentic. (Adams, Robert M., *Utopia,* pp. 108-113)

Giles, who supervised the printing of the first edition (Louvain, 1516), wanted "to add further to the fun" and designed a Utopian alphabet and even wrote a poem using its characters: (Monti, p. 96)

The Utopian alphabet (shown at the very top of the page) is actually Latin, with the letter characters changed into circles, curly semi-circles, a triangle, right angles and squares, some with additional marks.

The middle section of the page is entitled "**A Quatrain** [4 line verse] **in the Utopian Language.**" It is written in Utopian letters, with the transcription above, which appears to be a strange mix of Latin and Greek.

To clarify, Giles has written underneath: "The literal meaning of these lines," and has composed 4 lines in Latin which translate:

Utopian Alphabet
and encoded words
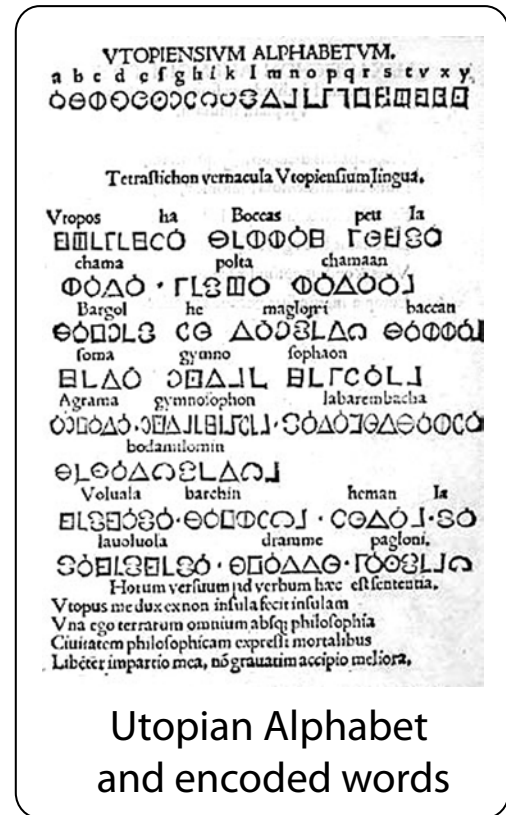
When I was not an island, the commander Utopus made me into an island.
I alone of all nations on earth, without philosophy,
Have presented to mortals a philosophical state
Freely I share what I have, not unwillingly I accept what is better.

(Miller, *Utopia*, p. 2)

Richard Firmage, in his 1993 *The Alphabet Abecedarium,*
writes that alphabets like this Utopian Alphabet were:

"Never intended for general acceptance or even any actual use,
however, being merely literary appendages or embellishments...
These letter forms are literary conceit or exercise in ingenuity,
meant to give a flavour of authenticity to fictional accounts
of the civilization of imaginary societies."

(Firmage, p. 226)

Geofroy Tory, in his 1592 *Champ Fleury,* devoted a full page to the Utopian alphabet. He claims they were written my More himself, who he calls "Morus Langlois" (More of England):

**"These are letters which we might call
Voluntary letters, made at one's pleasure,
as are those which the makers of ciphers
and decipherers drew in such shape and form as they chose,
to compose new things, which cannot be understood
without knowing the alphabet of the said Voluntary letters**."

In other words, these letters are like a code. Dee owned Tory's *Champ Fleury*, and its influence is evident elsewhere in the *Monas*. (For example, the "Arbor Raritatis" diagram, the letter "I" being the basis for the geometric construction of all the Latin letters, etc.) I'm not suggesting that Dee took the Utopian alphabet seriously, but that he used it to hint about his own alphabet code. Here is Tory's presentation of the "Utopian and Voluntary Letters":

Giles' alphabet only had 22 letters, but Tory includes a letter to represent that missing "Z." It's pretty evident that the first 11 letters are made from circles and semi-circles, the last 11 letters are made from right angles and squares, but the middle letter M is an equilateral triangle (curiously, like Dee's signature).

Sir Thomas More was an inspiration to Elizabethans not because he was a martyr, but because his life was ruled by his own conscience. In the Corollary to Theorem 22, Dee addresses,

**"those in whom inwardly there blazes a fiery vigor...
May they readily lend an ear to the great Democritus..."**

We've seen that Democritus was one of the original Greek "atomists." We've also seen that Erasmus called Thomas More "Democritus" in the introduction to *In Praise of Folly*. Democritus felt there was a state in which the soul can live peacefully, with no fear or superstition – an ultimate cheerfulness – and has been called the "laughing philosopher."

But there's something else about Democritus – his name includes an M and a T.
Dee writes that "Democritus" announces that,
"this Doctrine is not Mythical but Mystical."
Two more words containing M and T.

Tory's Version
of the
Utopian alphabet

166
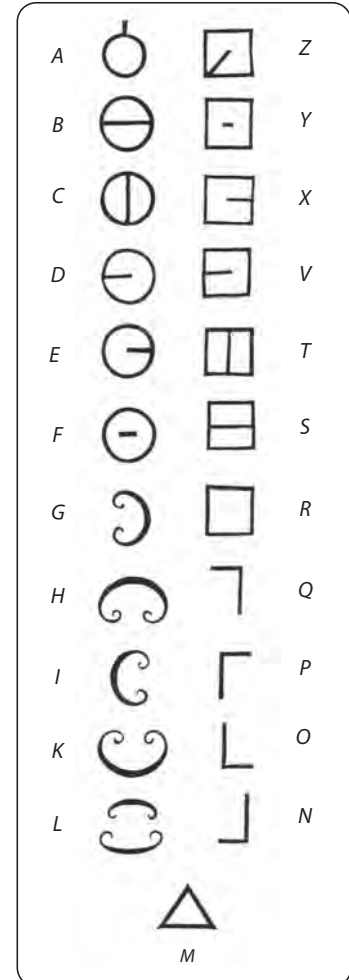
What does Dee mean with all this M and T business?

The easiest place to see what they mean
is in the Monas symbol itself.
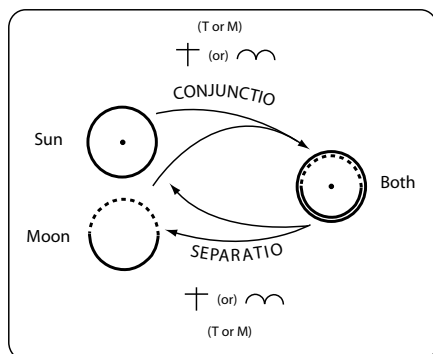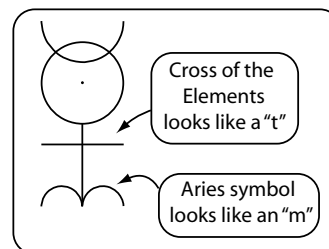The Cross of the Elements looks like a "t."
The Aries symbol looks like an "m."

Cross of the Elements looks like a "t"

Aries symbol looks like an "m"

As Dee tells us in Theorem 4, the Moon half-circle emulates the Sun circle and "**she even becomes Transformed into him**."

My summarizing illustration for Theorem 9 shows the Sun and Moon in the continuous process of "Conjunctio and Separatio."

(T or M)

✝ (or) ⌢

CONJUNCTIO

Sun

Both

Moon

SEPARATIO

✝ (or) ⌢

(T or M)

To which part of the process do "**T**"refer?
And what about M?
The answer seems to be:
They both refer to both parst of the process.

"T" can refer to either separatio or conjunctio:
The "T" is somewhat like the letter "X."
Dee explains in Theorems 6 and 20
that the "X" can be seen as
Ternary
(like conjunctio, two lines and a common point)
or as Quaternary
(like separatio, 4 lines).

So if "X" can be seen as both,
perhaps "t" can as well.

The Moon and the Sun...

...are in a continuous process of Separatio and Conjunctio
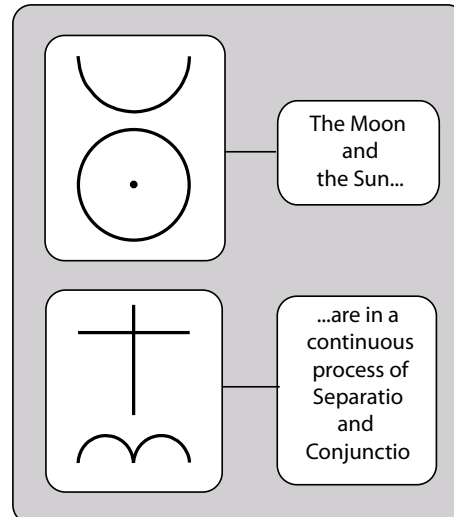
"M" can refer to either separatio or conjunctio:
The 2 horns of Aries seem to be
two half-circles (separatio),
but in Theorem 21, he shows them as
"closed up" into a circle (conjunctio).

What did Dee mean by "Grammarian"?

Throughout the Monas, Dee refers to "**grammarians**." This word makes me think of "English teachers" and "past participles." We call Elementary Schools, where we learn to read and write, "Grammar schools."

But even grammarians have a hard time defining "grammar." Let's go to the root of the word "grammar" to see what it meant to the Greeks.

The Greek word *gramma* comes from the word *grapho* which originally meant "**to scratch**," as in scratching marks on a tablet for counting things. Then, it morphed into "**drawing lines**." Then, it morphed into "**drawing characters like letters.**"

So in Greek, *gramma* means "**that which is drawn or written**" or "**a letter**" (or, in plural, "**the alphabet**.")

A Greek, a *grammatikos* is someone who knows his or her letters well, or a "grammarian." To the Greeks, grammar meant philology (love of learning), which includes both literary history and language structure. In the Middle Ages the meaning of the word "grammar" became restricted, referring to just the "language structure" part.

In Early English, "grammar" meant only Latin grammar, as Latin was the only language taught grammatically. In the 1500's, grammar only started to include English grammar.It wasn't until around 1600 that Ben Jonson wrote the brief treatise *The English Grammar*.

So, in Dee's time, a grammarian was anyone who understood **Latin** texts, sentences, words, and ultimately letters. To Dee, a good grammarian understood the texts, sentences, words, and letters of Greek and Hebrew as well. Grammarians were not "English teachers."

In his search of how Nature works, Dee boiled sentences down into words, words into "etymons" (true roots), and even "etymons" into letters (and even letters into their divine essences: points, lines, and circles.) Language experts still use the word "**etymon**," meaning "a word or morpheme from which a later word is derived."

A morpheme is the smallest meaningful unit in the grammar of a language. The word "unkind" has two morphemes, "un" and "kind." (But, morpheme doesn't necessarily refer to one syllable. For example, the whole word "mahogany" is one morpheme. It can't be broken down into smaller meaningful units.)

Dee's fascination with word stems and word origins can be seen in his *Preface to Euclid*, where he "would gladly shake" the "**earthly name of Geometrie**" (earth-measurement) for **Megethalogia** (a study of magnitudes).

Also, the he coined the names of the 19 "Sciences, and Artes Mathematical," by compounding Greek root words, for example: "**Helicosophie, Hydragogie, and Pneumatithmie.**"

TEM and MET words

Like his hero Plato, Dee loved etymology. Plato dedicates over half of his *Cratylys* to explanations of various etymologies (word origins). Even though modern scholars have determined that Plato was wrong about some of them, *Cratylys* is a testimony of how important study etymologies were to him.

Dee saw words which included the letters T and M as describing things which involve the continuous process of conjunctio and separatio.

For example, the Greek word "**temein**" means "to cut" (much like separatio).
A "**tomos**" is a cut off section of a long book, or a tome.
A "**tempus**" is a portion cut off.
A space that was "cut off" for sacred purposes was a "**templum,**"
from which we get the word temple.
A time that was "cut off" was a "**tempestas,**"
from which we get "tempo" and "temporary."

The Greek word "**tmesus**" means a cutting.
In geometry, a "**tmema**" is part of a circle cut off by a chord.
Mount **Tmolos** (now Timilous) is a tall ridge
that "cuts" the plain in ancient Lydia
(where King Gyges mined his gold or gold alloy electrom.)

Another branch of Tem became "tomo," to cut that lead to "**tomeus**," a knife
and "**atom**" – that which cannot ("a" means "not") be cut into anything smaller.
We also get the word "**epitome**," to "cut down" into a summary,
and "**lobotomy**," archaic surgery that cuts the prefrontal lobe of the brain.

However, "tem" words can also signify conjunctio instead of separatio.
The Latin word "**tempero**" means "to mix in due proportion or moderation."
From this root we get words like "temperate,"
"temperature" (a "conjunction" of hot and cold),
"temperament"(no, it's not spelled temperment),
and even "tempered" steel.

The Greek prefix "**met**" means among, sharing,
or having an action in common,
so it's much like "conjunctio."
This meaning morphed into "change," as in "metamorphosis"
and "metathesis" (a transposition or change of sides).
One might even add the Latin word "mixtus," meaning to mix, from which we get mixture, a "conjunctio" of several things.

To summarize,
Dee saw the letter X as a symbol of the
whole process of the "union of opposites."

This process has two parts, separatio and conjunctio,
which he saw as T and M
(or as M and T)

To Dee, words that contained an M and a T
described things in which this
flow of "separatio and conjunctio" could be found.

Based on modern etymological studies,
Dee's word origins might not be technically correct,
(as he lived before the discovery of Proto-Indo European Language).

But what's important is that this is how Dee saw it,
and how utilized it in his clues in the *Monas*.

It seems as though, to Dee,
any word containing T and M,
in any language,
was eligible to be seen
as involving "separatio and conjunctio."

Let's keep an eye peeled for more examples.

Later, you'll realize that this has been an important diversion,
but now, let's **see** another important way that Dee **envisioned**
the cuboctahedron, made from its 4 pairs of Bucky bowties.

Following Alberti, Dee used the idea of "Visual Pyramids" to describe how vision works

The medieval concept of vision derives from the Alhazen's works on what the Italian Renaissance artists later referred to as **"perspectiva."**

In his *Preface to Euclid*, Dee lists the Art of Perspective
first
among all of the important Mathematical Arts
which derive from Arithmetic and Geometry.

And he gives three good reasons why:

First,
Without understanding Perspective,
"**Astronomical Appearances**"
(the sun, moon and celestial objects)
cannot be easily comprehended.

Second,
"Perspective" deals with **Light**,
"…the First of God's Creatures."

Third,
it deals with the **Eye,**
"the light of our body,
and his Sense most mighty,
and his organ most
Artificial and Geometrical"
(Artificial, meaning skillful,)

Dee says Perspective deals with
"Direct, Broken, and Reflected Radiations."

Understanding "**Direct Radiation,**"
we might realize how our eye is "deceived"
when a round Globe or Sphere, some distance away,
appears to us a "flat and plain Circle."
Or given two objects moving at the same rate,
why the closer object appear to be moving faster.

"**Broken Radiation**" explains why an oar in water
appears to be bent or "broken" due to refraction.

The study of "**Reflected Radiations**" includes not just flat mirrors,
but convex and concave ones as well (the science of Catoptrics).

Dee cites the concave lens given to him by Sir William Pickering

in which the image appears reversed,
(like looking at your face in the bowl of a shiny spoon).
Dee's diary tells of the fun he had showing Queen Elizabeth
and her courtiers this mirror when they visited Dee at Mortlake.
When the courtier thrust toward the right side of the mirror with a knife,
his reflection thrust towards the left.

(Dee, Preface to Euclid, p. b.j. and b.j. verso).

In 1558, Dee had gathered together research done by various authors
who had written about Catoptrics in a 5-volume book called "*De speculis comburentibus*."
It is illustrated with many intricate drawings of how light reflects off of flat, concave,
and convex mirrors, as well as the Euclidean geometry of conic sections.
However, the text was never finished and the work was never published.
Dee's hand-written manuscript is now resides in the British Library
and his experimental notes, "*Experimentia in Speculo*" are in Oxford's Bodleian Library.

(MS Cotton Vitellius C, Vii, art 5 and Bodleian Library MS Sloane 3854, *Experimentia in Speculo*, ff 76r-80y)
(letter by Jose Rodriguez at : http://www.levity.com/alchemy/a-archive_nov00.html)

Zography means "painting from life"

Closely related to the Mathematical Art of "Perspective"
is another Mathematical Art that Dee calls "Zographie."
(which I like to modernize to Zography).

In Greek "Zo-" refers to "life,
and "-graphy" refers to " drawing or writing,"
so Zography is simply:
"painting from life."

It's pretty clear that Dee derived his ideas on how vision worked
from Leon Battista Alberti's 1435 treatise "On Painting' (De Pictura).

Dee's (1570) definition of "Zography" is quite similar
(practically verbatum)
to Alberti's (1435) definition of "a painting."

Leon Battista Alberti
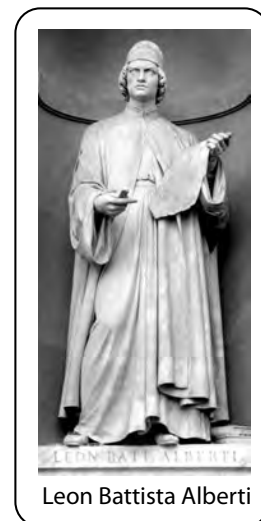
**Alberti's (1435)
definition of a painting**

"A painting is the intersection
of a visual pyramid at a given distance,
with a fixed centre and a defined position of light,
represented by art
with lines and colours
on a given surface."

**Dee's (1570)
definition of Zography.**

"Zographie, is an Arte Mathematicall,
which teaches and demonstrates, how
the Intersection of all visual Pyramids,
made by any plain assigned,
(the Centre, distance, and light, being determined)
may be represented by lines, and due proper colours."

Alberti admits that his treatise does not deal with the question of whether vision occurs on the surface of the eye "as on a living mirror" or at the juncture of the inner nerve. Likewise, he avoids discussing the actual nature of light. Instead he takes a geometrical approach to explaining eye perceives objects in its field of vision.

Alberti called all the rays coming from the outline
(boundary or edge) of an observed object
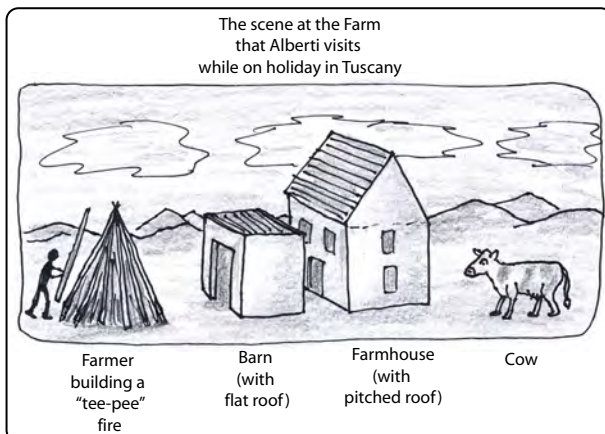**"extrinsico"**
(extrinsic or extreme).

All those issuing from the area within the edges
were called **"intrinsico"**
(intrinsic, median, or inner).

All the **"extrinsic rays"** combined
form the **"visual pyramid."**
For example, if you looked straight-on
to a square-shaped object,
the extrinsic rays would form
a "pyramid with 4 sides and a base."
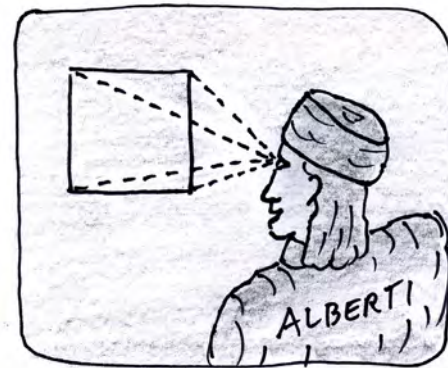
The "visual pyramid"
from a square -shaped object
is a sideways "Pyramid of Giza" shape

Nowadays, the word "pyramid" is generally envisioned as the
"4 sides plus a base" kind, like the enormous Pyramids in Giza, Egypt.
But, a geometric pyramid can have 3, 4, 5, 6, 7... or any number of sides,
all of which taper from a base to a single point at the apex.

The scene at the Farm
that Alberti visits
while on holiday in Tuscany

Farmer building a "tee-pee" fire | Barn (with flat roof) | Farmhouse (with pitched roof) | Cow

Imagine that Alberti was on holiday,
visiting a Tuscan farmhouse,
which had a flat-roofed cow barn.
The farmer was preparing to celebrate
by building a huge tee-pee-shaped fire
to ignite later that evening.

Viewing the scene face-on, Leon Battista Alberti would see the...
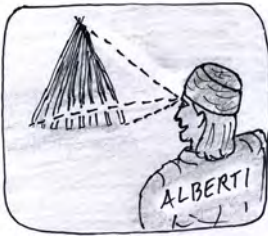
... wood for the fire
as a
3-sided pyramid
(plus a base)

... the barn
as a
4-sided pyramid
(plus a base)

... the farmhouse
as a
5-sided pyramid
(plus a base)

... and the cow
as a
many-sided pyramid
(plus a base)

It's clear that the **simplest** pyramid
is the "3-sided pyramid plus a base."

One might suggest that a pole in the ground that
looked at a **line** might be simpler.
Geometrically-speaking, a line has no thickness,
but a pole, even a thin rod, has a certain thickness.
Thus, to the viewer, a pole is just a tall, thin
4-sided pyramid (plus a base).

The "visual pyramid" from a pole or stick
is actually a tall, thin four-sided pyramid
(plus a base).

Note that all these examples of "visual pyramids"
have triangular sides. Some are pretty skinny,
but they all taper to the apex point, the eye.

However, only the **simplest** visual pyramid
also has a triangle as a base,
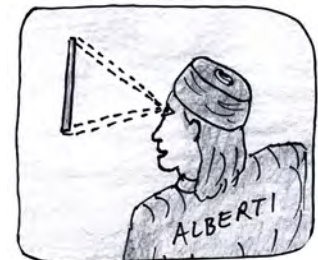making it a tetrahedron (four-sided).

If the length of one side of the base
equals the distance from the base's corners to the eye,
the "visual pyramid" would be a "regular tetrahedron"
(comprised of 4 equilateral triangles).

Alberti explains that the **intrinsic** rays
are the ones which transmit information to the eye
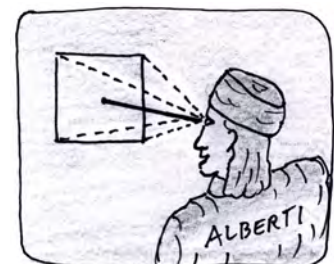about **surface** qualities, like light, shade, texture and color.

He says they behave
"like a chameleon, an animal which takes to itself
the colors or things near it."

Amon the rays, there is **one intrinsic ray** that is the
"**most active and strongest of all the rays...
which merits the name, prince of rays**."
It's called the **centrico** ray [centric or central],
and it goes from the geometric center of the object
to the viewer's eye.

The one "intrinsic" ray emanating from
the exact geometric center
of the face of the object observed
is called the "centric ray"

(Alberti, *On Painting*, pp. 47-50 and Kemp, *The Science of Art*, p. 22).

Using a "virtual" camera obscura
to show "cross-sections"
of two tip-to-tip tetrahedron.

Like Alberti, Dee would have been challenged to explain
how the image in a camera-obscura eyeball got "corrected"
(so we don't see everything upside down and backwards).

But, he did have a firm grasp on the geometry
of the behavior of light in a camera obscura.

To make it easier to explain,
I've designed a "virtual" building
with two square rooms
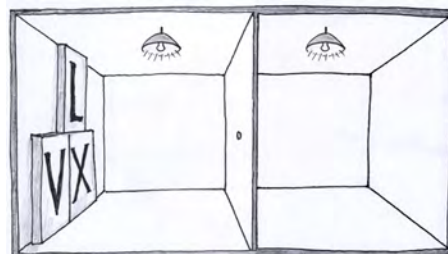and a small hole in their common wall.
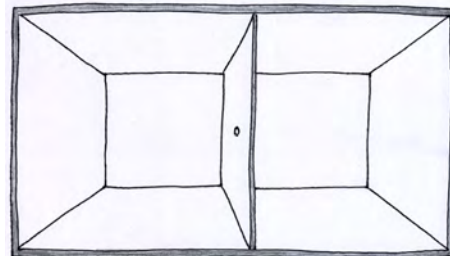
In the left room I've arranged 3 letters,
printed on large thin boxes,
(cleverly spelling the Latin word for light).

When a bright light is turned on in the left room,
an inverted and reversed image
will appear on the wall of the dark room on the right.
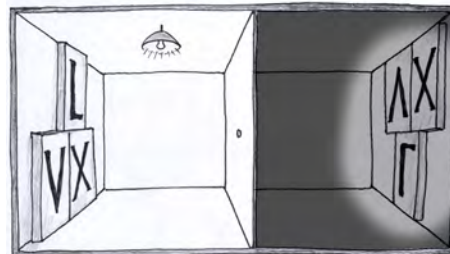
If a bright light is turned on in the **right** room,
the image will "disappear."

But, turning that light in the right room back off,
the image will "reappear,"
demonstrating that it had really
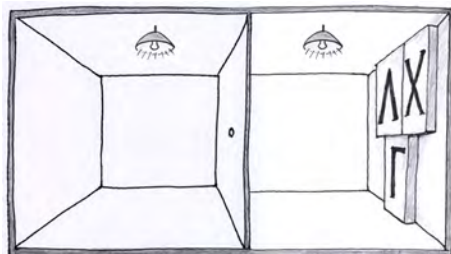been there the whole time.
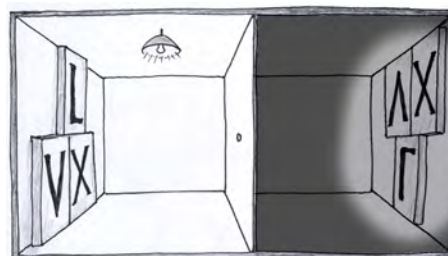
For this next series, I have deleted the 3 letters from the left room
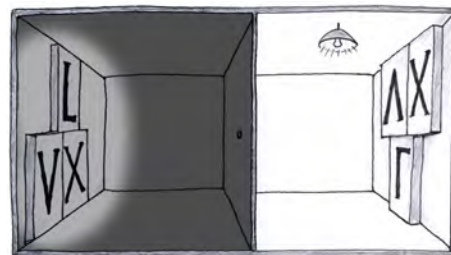and made 3 new letters for the right room.

(Actually I could reuse the V and the X, but the L is painted on intentionally "backwards,"
looking like a Greek capital letter Gamma. I'm still implying the word LUX, not GUX).
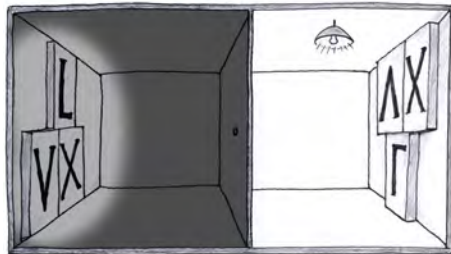
Turning the light on in the right room
makes a camera obscura image
appear in the left room.

Turning the light on in the left room,
the image disappears.

Turning that light off again,
the image reappears.
Again, it was there the whole time.

Now, if we set up
the 3 "LUX blocks" in both rooms,
and illuminate both rooms,
we should realize that camera obscura images
are projected in **both** directions.

(even though we can't see them
because the lights are on)
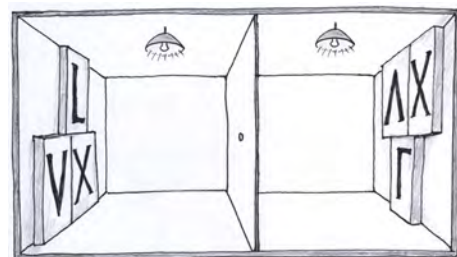
Geometrically,
we might describe what is happening here,
as **two tip-to-tip tetrahedra**!
Note how the rays crisscross
as they pass through the central hole.
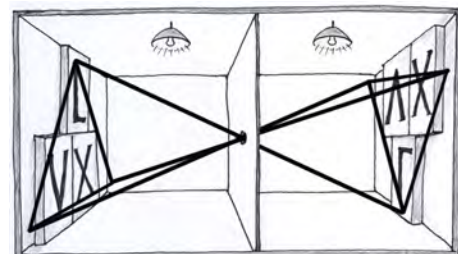
For this next demonstration,
let's remove the 3 blocks from the right room.

Instead, let's set up a large white board vertically,
a few feet from the wall.
The projected image appears to have shrunk.
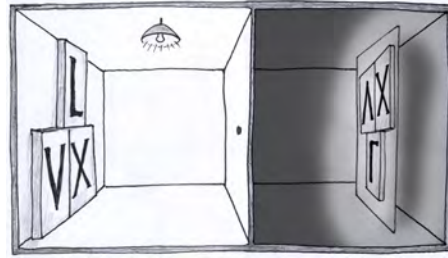
Next, we'll move the board
into the center of the room.

Now, the image is even smaller.
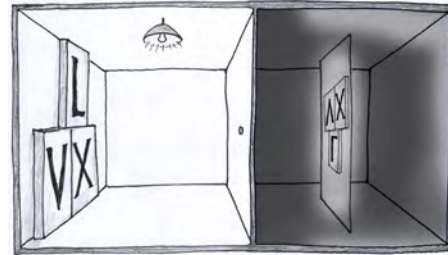
Finally, we'll move the board
to a few feet from the hole.

The projected image is now very small.
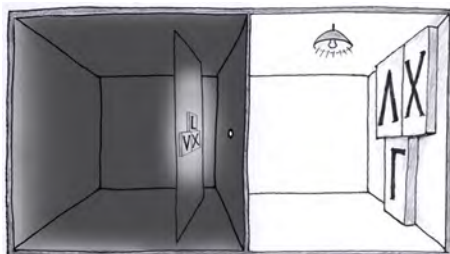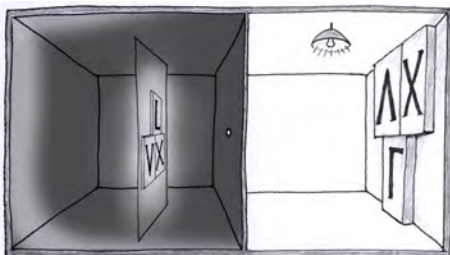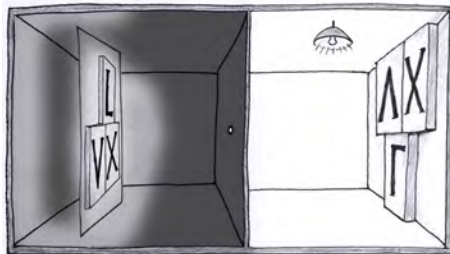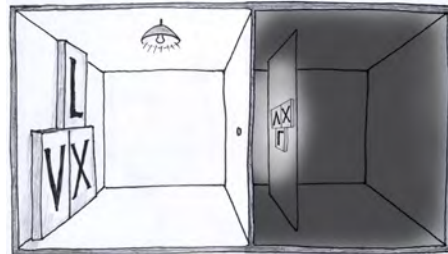
Obviously the same thing will happen
going "in the other direction."

The image will get smaller and smaller
as the board gets nearer to the hole.

Next,
we'll envision all the boards
at the same time.
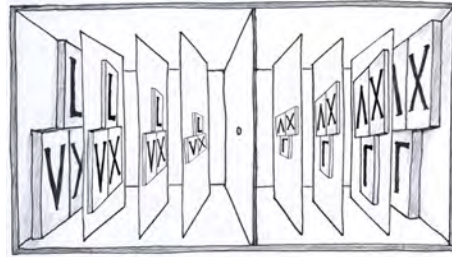
Now we can get an
even better feel for why the
behavior of light in a camera obscura
is is best depicted as
two tip-to-tip tetrahedra.
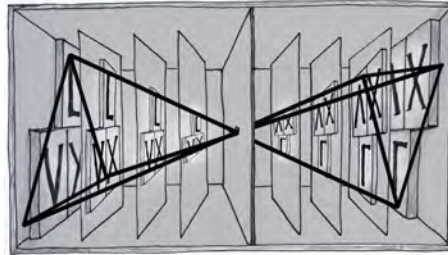
Eliminating the boards,
the "cross-sections" of the tetrahedra
are even more evident.
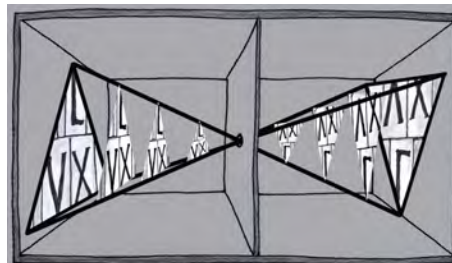
So where are all these cuboctahedra?

If cuboctahedra are Nature's Law,
why don't we find them while taking a walk in the woods?

Apparently we don't see them because they're so prevalent.
They're everywhere.

They surround every **point** in the Universe.
The camera obscura is a tool to help us realize they are there.

Imagine any ordinary point.
Here is one about 6 feet above the ground,
in the middle of the desert.

Imagine a random point
in the middle of the desert

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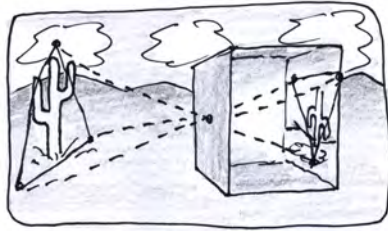
That point seen as a
camera obscura
aimed to the west

If we built a camera obscura room
just to the east of the hole,
we could see the view to the west.
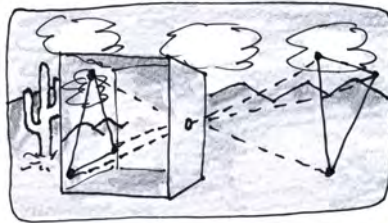
That same point
as a camera obscura
aimed to the east

Or if we built a camera obscura room
just to the west of the point,
we could see the view to the east.
Now it's easier to see
one pair of tip-to-tip tetrahedra
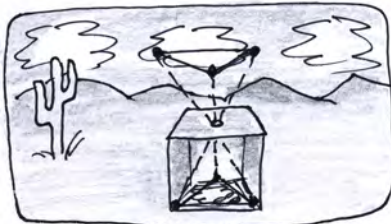
That same point
as a camera obscura
pointd upwards

Similarly, we could build a camera obscura room
under that point
and see blue sky and billowing clouds
projected on the floor.
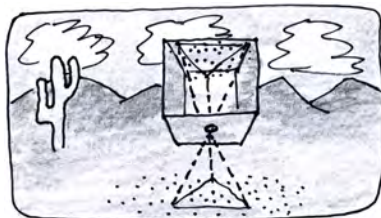
A "suspended" camera obscura
aiming down to the sand

We could suspend a camera obscura
above the point,
we would get a projected image of sand.

Thus, we have identified
a second pair of tip-to-tip tetrahedra.

There are thousands upon thousands of ways
we could orient the camera obscura,
but the minimal assemblage needed to define that point
would be 4 pairs of tip-to-tip tetrahedra
arranged as a cuboctahedron.
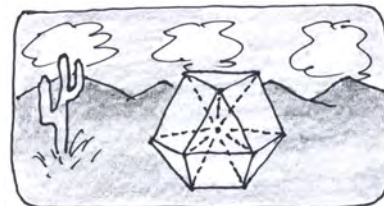
The minimum assemblage needed
to define that point is the
8 tetrahedra arranged
perfectly as a cuboctahedron.

The camera obscura is a tool to help us see the gazillions of cuboctahedrons
that define the gazillions of points in space.
This concept of so many overlapping cubotahedrons everywhere is hard to grasp.
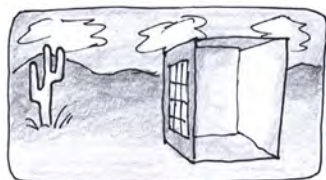
One way to help comprehend it
is by playing with a camera obscura.

Imagine a small room with one large window.

Diffuse light pours in through the window
and bounces off all the walls,
giving the interior a nice soft overall illumination.

The diffuse light coming through
a window is actually thousands
of overlapping camera obscuras

But that "soft glow" is actually thousands
of overlapping projected images
coming through the thousands of "holes"
that make the whole window.
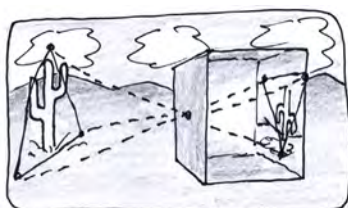
That window reduced
to one small hole

To demonstrate this, let's reduce
the window down to only one small hole.
An image is projected on the opposite wall.
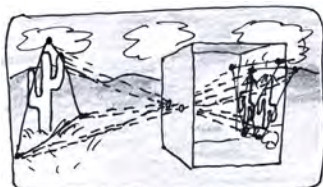
Camera obscura with two holes
and two overlapping
projected images

If there are two small holes,
two images are projected on the wall.
The images appear "off register."

They are not aligned with each other.
It's like double vision.
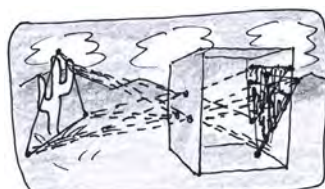
Camera obscura with three holes
and three overlapping
projected images

If there are three holes,
there are three overlapping projected images.

If there are many holes,
the many "off register"
projected images
start to "blend together"
into one general glow
of diffused light.
To conclude,
camera obscura images
are eveywhere,
(always have been,
since light was first created.)
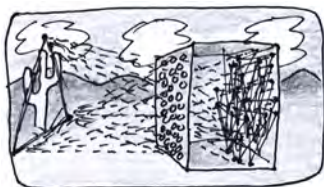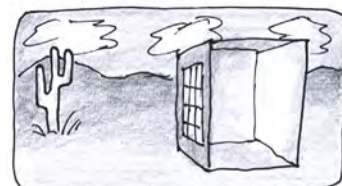
Camera obscura with many holes
and many overlapping
projected images

The diffuse light coming through
a window is actually thousands
of overlapping camera obscuras

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Dee's
cryptic references
to the
camera obscura
in the
Monas Hieroglyphica

Having explored Alberti's (and Dee's) understanding of how vision works,
and the most economical geometric description of how a camera obscura works,
we can now discern more clearly what what Dee is saying in his Letter to Maximillian.

Dee explains how the concepts *Monas Hieroglyphica*
will benefit 14 different professions (in different ways).

There is a sequence of 3 of them,
to the **Astronomer**,
to the **Optician**,
and to the **Expert on Weights**
that all seem to refer to the camera obscura.

Dee's Advice to the Astronomer:

**And won't the ASTRONOMER regret all his sleepless vigils
and cold labors he has suffered under the Open Sky,
when here, without any Discomfort from the Air,**

**Under his own roof, with windows and Doors Shut on all sides,
at any given Time, he is able to observe the movements of the heavenly bodies?**

And, indeed, without any Mechanical Instruments made from Wood or Brass?

The phrase "windows and doors shut on all sides" is another way of saying "a dark room."
"Without any wooden or brass mechanical instruments"
rules out the use of an orrery, a mechanical model that simulates celestial movements.
During the day the solar disc traces out the path of the sun.
At night, the lunar disc (or crescent) can be seen projected inside the dark camera obscura room.
Granted, its hard to see the projection of stars and planets in a camera obscura without a lens,
but Dee doesn't mention anything about not using glass, only wood and brass.

Next, Dee gives advice to the "Perspectivus," or the Optician.
Nowadays, an Optician dispenses glasses and contact lenses.
In Dee's day, an Optician was the person who actually
made optical equipment like lenses and mirrors.

Dee's advice to the Optician:

**And won't the OPTICIAN condemn
the Senselessness of his Ingenious work,
laboring in all sorts of ways to make a Mirror
according to a Line (appropriately curved in a circle)
of a Parabolic Section of a Cone,
which will attack any Matter (able to be burned by fire)
with the incredible Heat from the Rays of the Sun.**

**Yet here a Line is presented,
resulting from a Three-Cornered Section of the Tetrahedron,
from which, when Made Full-Circle, a Mirror may be found that
(even when the Sun is being blocked by Clouds)
can reduce any kind of Stones or Metal into Impalpable Powders
by the force of (truly the very strongest) Heat.**

In Dee's time, Opticians were struggling diligently
to determine the optimum curve for a parabolic mirror
that could be used to project a focused image
(or even focus the sun's rays to make powerful heat source).

Instead they could simply use a camera obscura.

On a cloudy day, the solar disc would not be projected,
but the image of what's "outside" is still projected.
Even solid objects like stone or metal (like stone walls or cannons)
are projected on the inside wall.

You can try and grab the stone or metal objects in your hand,
but they are like an "Impalpable Powder"(lighter than a feather).

These things are visible bcause of the light from the sun (even if it's diffused by clouds).
Dee refers to the sun as "the force of (truly the strongest) Heat."

To really understand how Dee describes the camera obscura,
let's look closer at part of the primary source,
his original Latin text.

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Dee's Original Latin Phrasing
"…Cum, hic ex Tetrahedri Sectione Trigonica,
Linea exhibeatur;
ex cuiss Forma Circulata,
fieri potest Speculum…"

C. H. Josten's Translation
"…yet here a line is revealed
as resulting from a trigonal section of the tetrahedron
after whose shape, when rendered three-dimensional
a mirror may be formed…"

My Translation
…Yet here a Line is presented,
resulting from a Three-Cornered Section of the Tetrahedron,
from which, when Made Full-Circle,
a Mirror may be found...

... a Three-Cornered Section of the Tetrahedron...,

Trigonica, in Greek means "three-cornered." In other words, a triangle.
In my earlier demonstration of the "virtual camera obscura rooms,"
these triangular "cross-sections" of tetrahedra are clearly evident.

...a line presents itself...

Picture the right-side room as a giant eyeball,
and the left-side room as a visual pyramid,
whose base is the triangle of letters.

The "line which presents itself"
is Alberti's "**centric ray,**"
emanating from the middle of the triangle
(where the blocks of L, U, and X all meet)
and extending to the apex of the tetrahedron,
at the pupil of the eye.
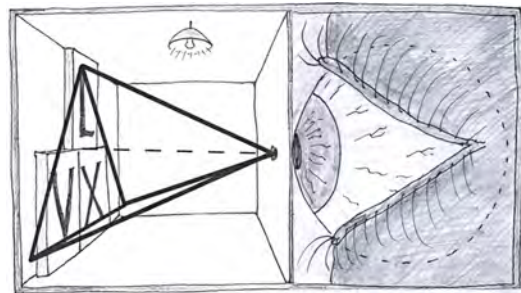
... when Made Full-Circle...

Dee's term "**Forma Circulata**" is a tricky one, but it is quite revealing.
It seems suggest the formation of a circle, or perhaps even the form of a sphere.
But in this context, I think Dee is using the phrase "in Circular Form" metaphorically,
in the sense of: "make complete,"
"make whole,"
"come full circle,"
"round out,"
or
"complete the circuit."

In this geometric-optical demonstration, the visual-pyramid-tetrahedron in the left room
is "made complete" or "come full circle" when mated with its opposite,
the "projected" visual-pyramid-tetrahedron in the right room.

One tetrahedron
(like a visual pyramid
looking "outside"
a camera obscura) ...

implies another
tetrahedron
(like the visual
pyramid inside a
camera obscura)

...when it is in
"Forma Circulata"
(complete, whole,
or in "Circular Form")
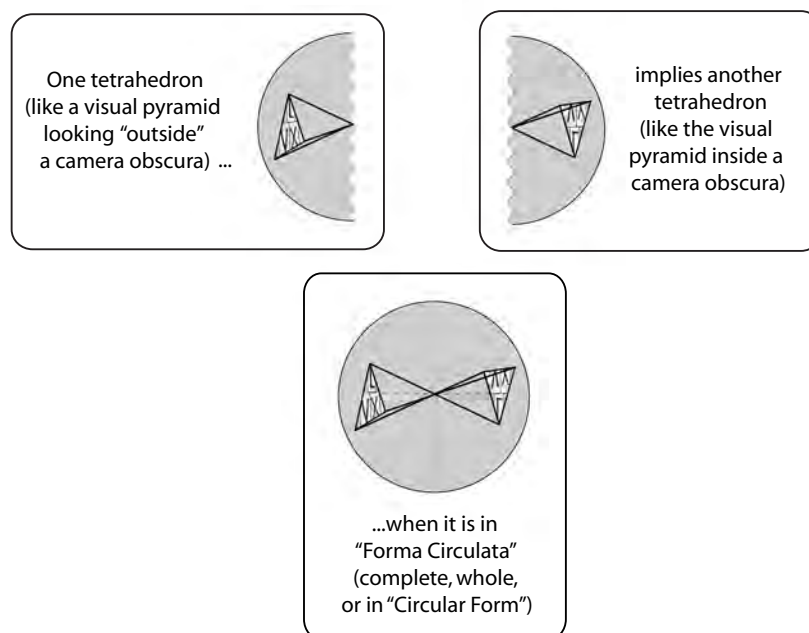
Dee tells us that when all this happens, "a mirror is found."
Again, Dee is speaking metaphorically.

He's not using mirror in the sense of the "shiny silvery surface that reflects,"
but in the sense of a copy, an imitation, a simulation, or an echo.

He's using the term mirror (*speculum*) metaphorically to describe that
incredibly accurate, full-color, live action, projected image made in a camera obscura.

It really is much like a "mirror-image," a replication of every
tiny detail in the "scene" on the other side of the aperture.

That centric ray also extends straight
through the hole to the exact center
of the projected image.
(Incidentally, along its path it passes
through the exact center of both of the
3-D tetrahedrons and their common point.)
How all these light rays cram through
that teeny hole and come out the other side
so nicely organized seems magical.
But, when it is explained geometrically,
it's not all that mysterious,
as this visual summary chart shows:

...a Line
presents itself...

...When this is
Made Full-Circle, ...

From the
3-Cornered
Cross Section
of a
Tetrahedron...

...a Mirror
can be
found..

Dee's cryptic description
of a camera obscura,
written as part of
his advice to Opticians,
in his *Letter to Maximillian*
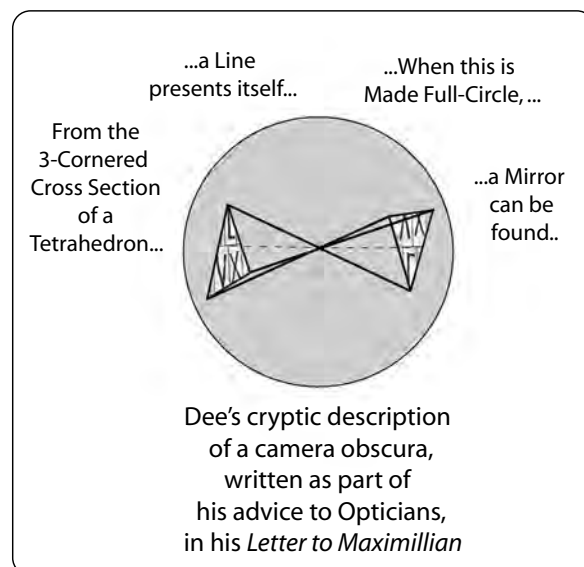
Next, Dee advises the "Expert on Weights" what he can learn from the "Magistery of our Monad."

Dee's advice to the Experts on Weights

**And will not be, who has devoted all the Time of his life
to making exacting measurements with WEIGHTS,
judge just how well his Labors and costs have been invested,
when here, the Magistery of our MONAD will teach him,
most assuredly by actual Experience,
that the Element of Earth can float above that of Water?**

This preposterous claim seems to be some
obscure alchemical concept about the Elements
but actually, Dee is simply playing with the idea
that the image in a camera obscura is inverted.

For example, if Dee looked out the west-facing window of his study,
he might have see his children playing in the dirt along the riverwalk.

Beyond that was a large expanse of the Thames River
as it makes its great bend at Mortlake.
If he blocked up the window (except for a small hole),
the scene outside would project
onto the walls of the study upside down (and reversed).

(WATER ABOVE EARTH)   (EARTH ABOVE WATER)
JOHN DEE'S STUDY AT MORTLAKE
AS A CAMERA OBSCURA
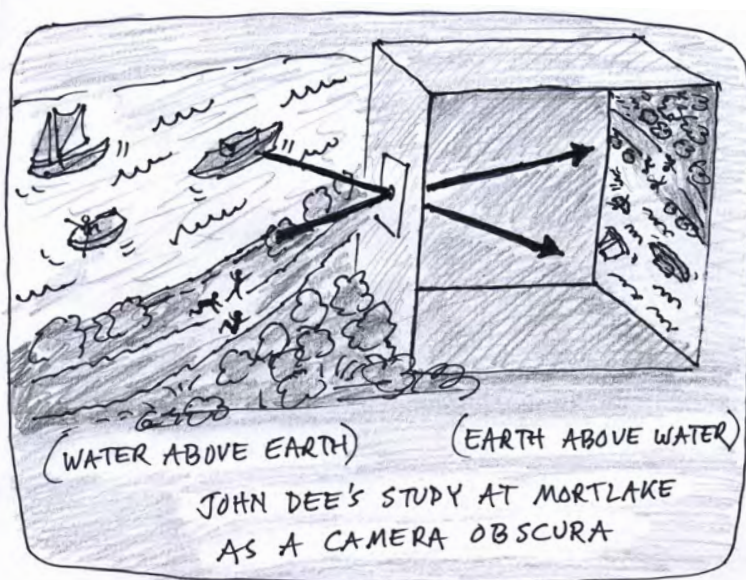
The image of children playing in the dirt (Earth)
would be projected **above** the image of the river (Water).
Thus, the idea that "**earth can float above water**"
is not preposterous after all.

Nonetheless, if some of Dee's less enlightened countrymen knew that Dee had sailboats, rowboats, and barges slowly moving back and forth across the walls of his study, and his children playing upside-down above that, they would accuse him of being some kind of magician and have him locked up (or worse).

Thus, Dee had to write cryptically about something that is really quite simple and natural.

Dee incorporated this "earth can float above water" idea in the John Dee Tower by directing the building crew to construct it on the crest of a hill overlooking water. I don't think it's simply a coincidence that the west window of his study at Mortlake and the west window of the John Dee Tower both look out over water.

Dee knew that when he crossed the ocean to visit, or perhaps even live, in the first Elizabethan colony on the John Dee River, every sunny afternoon the first floor room of his tower would be adorned with scintillating reflections off the water. He would feel like he was at home in his study at Mortlake.

Only this would be better. He would be in a round room.

And in a building he had proportioned harmoniously using "Nature's Laws," centrally located on the coast of what he perceived to be a land of opportunity and riches.

Dee cryptically refers to the Camera Obscura when explaining the Art of Zography

> Zographie, is an Arte Mathematicall, which teacheth and de-
> monstrateth, how, the Interfection of all visuall Pyramides, made
> by any playne affigned, (the Centre, diftance, and lightes, beyng de-
> termined) may be, by lynes, and due propre colours, reprefented.
> A notable Arte is this, and would require a whole Volume to declare the proper.

     Does Dee mention the camera obscura in the compendium of all the Sciences and Arts in his *Preface to Euclid*? Let's take a closer look at what Dee says about the Art of Zography.

     The *Oxford English Dictionary* calls the word "obscure" and the only reference cited is Dees' 1570 *Preface to Euclid*. Apparently it never caught on.

     But, Dee didn't make the word up by any means. Herodotus, and Plato both used the word to describe "**one who paints from life or from nature**." (Liddell/Scott, Zographos, p. 345)

     It's actual roots are the Greek words *zöos*, meaning "living things," and *graphos*, to draw. The Romans had a thing about the letter Z. They only used it in transcribing Greek names and words, but zography was not a word they chose to adopt.

Among the books in Dee's library was Marco Benavides Mantovas's 1566 *Zographia sive Hieroglyphica … naturae* (*Zography or Hieroglyphics of nature*). Marcus Mantova was a poet, not an artist, so he is speaking metaphorically about mental images of the world of living things. (Roberts and Watson, no. 1896)

The word *zography* is actually still used in Greece today. Dr. Slobodan Raicevic writes about the 15th and 16th Century "zographers" of Montenegro (now southern Yugoslavia) in his 1996 *Montenegrin Art of Painting*.

The "odd art" of Althalmasat

Dee describes how the skillful zographer can show you a "lively view of "summer's joy and riches" even in winter." He can depict "lying, creeping, sliding or swimming ... or flying creatures" or the "stars, sky, clouds ... even light."

Dee calls "Picture and Sculpture" "**sisters**" and recommends reading Giorgio Vasaris' 1550 *Lives of the Most Eminent Painters, Sculptors, and Architects*.

After discussing painting and sculpture at some length, Dee adds:

"a certain odd Art" called "Althalmasat"

> To thefe two Artes, (with other,)is a certaine od Arte , called *Althalmafat*, much beholdyng: more, then the common *Sculptor, Entayler, Keruer, Cutter, Grauer, Foun-* *der*, or *Paynter* (*&c*)know their Arte,to be commodious.

[In addition] To these two Arts (and others as well)
is a certain odd Art, called Althalmasat,
much more beholding than the common Sculptor,
Entayler [Intaglio artist]**,**
Kerver [Carver]**,**
Graver [Engraver]**,**
Founder [Moldmaker]**,**
or Painter (& etc.)
know their Art, to be commodious.

"Commodious" means beneficial or advantageous. What is this "odd art" that is so beneficial or advantageous to such a wide variety of artistic craftsmen?

Dee seems to have coined the word "Althalmasat" by combining three parts:
The prefix ***Al*** (an Arab prefix meaning "the"),
thalamus (a Greek word meaning room or chamber),
and the suffix ***at***.

Thalamos means "an inner room of a woman's apartment," a "bed-room,"
a "store-room,"the "hold of a ship,"
a "sacred chapel" or an "inner-most shrine."
It's related to ***thalame***, a "den or a cave."

Thalamus is much like the Latin word camera or "chamber,"
as in camera obscura (room + dark).
Inner rooms, caves, ship holds were naturally quite dark.
Dees' prefix "Al-" and suffix "-at" give it the sense
that it is the "Art of the" Darkened Room.

Dee recommends it to engravers and painters
because its a miniature version of a camera obscura
It would provide a image of a 2-D or 3-D subject
that could be scaled to any size, and then traced.

(For an in-depth explanation of the use of visual tools used in the Renaissance,
refer to Martin Kemp's *The Science of Art* or David Hockney and Charles M. Falco's *Secret Knowledge.*)

The Greek word *thalamus* is related to the Greek word *tholos*, a round building with a conical or domed roof, a vaulted chamber. The Greeks built a 60-foot diameter *tholos* in Athens to house the *prytanes*, or council leaders.

 Around 360 BC, they constructed one about 50 feet in diameter in Delphi. Vitruvius reports it was built by Theodorus the Phocian (several of its pillars are still standing today). Remains of a *tholos* in Epidarius suggest it was once one of the most splendid buildings in Greece.
(Vitruvius, *Book 7*, intro, p. 12)

The word *tholos* also refers to round or beehive shaped underground tombs built much earlier, from 1500 BC – 1100 BC. They were generally built into a hillside or covered with an earthen mound, but many were freestanding stone structures.

Many of them still exist today in Greece and in southern Italy (especially near Bari and Sardinia) where they are referred to as *nuraghe*.

(The *nuraghe* shown here are from just outside Bari near the beautiful Castel del Monte)

How can I be certain Dee knew about the connection between
thalamos (from which he named althalmasat) and *tholos*?
That's easy.
He drew a picture of a "tholos" on the Title page of the Monas!

If you look very closely at the circle representing
the Element Earth (on the left pedestal),
you can see a rounded, dome-shaped tholos,
There appears to be a with a person in front of it (probably for scale).
There is also a foreground,
a coastline (with a curious burst of light in the water).
and a mountain range.

With one hole for light,
the **thalamus,** inside a **tholos,**
becomes an **althalmasat**!

an illustration
of a **tholos**
on the
Title page
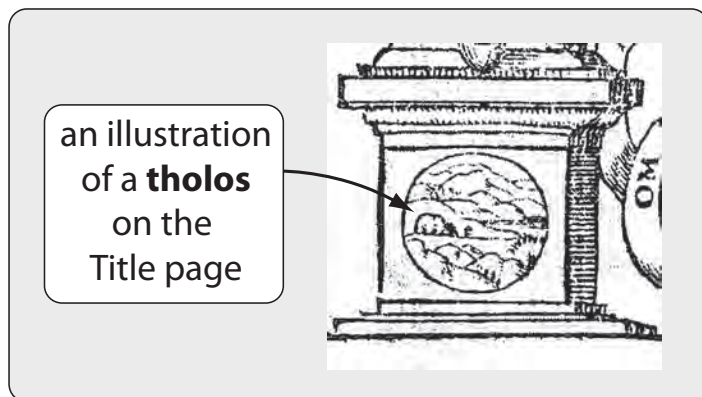
another example of a
"nuraghe" or "tholos"

Dee cuatiously refrains from elaborating on the odd Art of Althalmasat (part of the Art of Zography) in which "life" can be viewed in a "dark chamber.".The learned would be able to decipher his intent, but the vulgar were just as well kept in the dark (lest they accuse Dee of having ships seagulls, horsecarts and little people waltzing about his study walls).

When Dee synthesized all his knowledge about mathematics and optics
into this architectural masterpiece, the John Dee Tower,
he chose to make a "thalamos" (circular, domed structure)
with "althalmasats" inside (rooms that operated as camera obscuras).

It might even be said that the John Dee Tower
was the first movie theater in America.

Locus
Obscura

Recall that the Latin translations of Alhazen's *Perspectiva*
(of which Dee owned 5 copies)
used the term " **locus obscuras**" instead of "camera obscura.

"Locus" means a place, spot, room or place of abode.
"Obscura" means dark, obscure, indistinct.
So locus obscura means a "**dark place**."

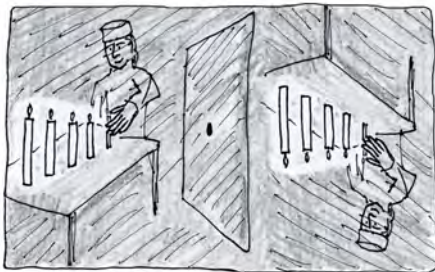Our modern-day word "camera" comes from the term "camera obscura" (room + dark or dark room) which was apparently coined by Johannes Kepler in 1604. Although *camera* means "room" in modern Italian, the original Greek and Latin word *camera* meant "a vault, arch, an arched roof, or anything with an arched cover."

locus obscura

Even though Alhazen's candle experiments were done in one large dark room, the lit candles were on one side, then there was a screen, and the other side was called the "**locus obscuras.**"
This dark place was not a separate room, so the word camera (room) wouldn't have even been appropriate. It's even less appropriate because Alhazen's "dark place" had nothing to do with a "vault, arch, or arched roof."

Dee (writing 40 years before Kepler's use of the term) was well-versed in Alhazen's *Perspectiva* and would have more familiar with the term "locus obscura."
And indeed he used this expression (cryptically) in the *Monas*!

At the bottom of the emblem following Theorem 24 he writes, "The Eye of the Vulgar will, here, be Obscured and most Distrustful."
In his original Latin, the word Oculus (eye) precedes the word "CALIGABIT."
The word "caligo" means "mist, vapor, fog, darkness, or obscurity" and is synonymous with obscura meaning "dark, dusky, or obscurity."


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Caligo comes from the Latin root word *cal-* meaning "cover." It is related to the Latin words *occulo* (to cover, with two c's), *clam* (hidden, secret, private) and *cella* (a store-room or chamber).

Though rarely used, the word "caliginous" (misty, dim, murky, obscure) can still be found in most English dictionaries. In 1794, Hester Lynch Poizzi wrote about "That caliginous atmosphere which fills London towards the 10th of November." In 1849, Edwrd Lytton wrote,"Her lone little room, full of caliginous corners and nooks."

Dee's Latin word "oculus" means "eye," but it doesn't take a jumbled-word genius to see the word "locus" among its letters.

OCULUS

LOCUS

Thus "…**Oculus CALIGABIT** …"
is Dee's cryptic way of expressing
"**locus obscuras**" or "dark place"
or what we call today a "**camera obscura.**"
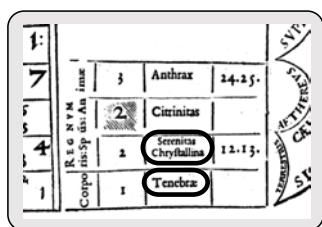
Furthermore, Dee uses a synonym for "obscura" in the "Thus the World Was Created" chart. The first stage of his alchemical quaternary is *Tenebrae*, or "darkness," which he uses metaphorically for the "black stage."

Just above it is the word *Chrystallina* or "crystalline," which he uses metaphorically for the "white stage." Recall that this word *chrystallina* is essentially the same word (*crystallinus*) used by the Latin translators of Alhazen to describe the lens of an eye. (from the Greek word *krystallos*, meaning "clear ice")

Dee adds the word "*Serenitas*," meaning "clear," above *Chrystallina*. He seems to be contrasting the idea of "clear and crystalline" with "dark and obscure."

These opposites relate to another pair of opposites found in the Monas, "Lux" and "Umbra," light and shadow. Dee thoroughly analyzes the anatomy of L, V, and X, in Theorem 16 and jokes in Theorem 17 that "**then a LIGHT (LUX) will appear.**" In the end of his Letter to Maximillian he uses the root word *umbra-* ten times on one page to subtly call attention to it.

(Dee, *Monas*, p. 9 verso)

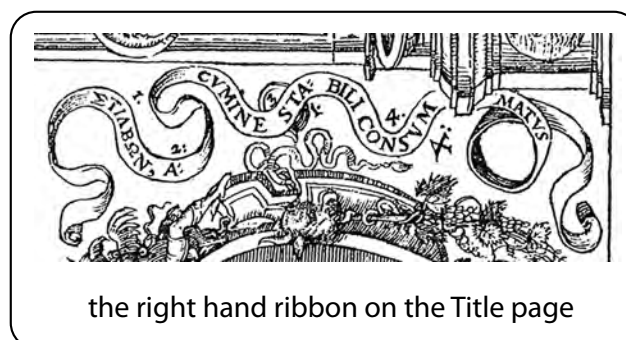To summarize, the behavior of light in a camera obscura is a main theme of the *Monas*, but Dee concealed it so he wouldn't be accused of practicing some kind of theologically incorrect magic.

But for those who explored optics scientifically, Dee's hidden terms "althalamos," "locus obscura," (in Oculus Caligabit) , "Chrystallina/Tenebrae," "LUX/UMBRA" would as clear as a sunny Spring day in London.

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The Title Page shouts out
"Camera Obscura"
(if you know the code)

Suspecting that Dee hid a reference to the camera obscura in the *Monas Hieroglyphica*, I focused my attention on the Title page.

The small hole on top of the shield of the central emblem seemed to suggest the aperture of a camera obscura. The tips of the two Mercury spears seem to be like light rays. They seem to meet at a point in the very center of that hole, just like the two tetrahedra meet at a point of vanishment in a Bucky bowtie.

I was studying the play of light and shadow on surrounding architecture, which further suggested the idea of "luminosity," when suddenly a light clicked on!

the right hand ribbon on the Title page

I had long wondered about
the numbers scattered
alongside the flowing ribbon
(on the right side of the emblem).
They seemed to refer to a word code
involving the letters of the words:
"STILBON ACUMINE
STABILI CONSUMMATUS."

My suspicion was that these 4 words
contained Dee's coined name for a camera obscura:
ALTHALMASAT.

There were several problems with my theory.
First, none of these words contain the letter H.
Second, Dee's little numbers (1, 2:, 3, 4, 1, 4:) sum to 15,
and there are only 11 letters in ALTHALMASAT.

Still, it felt like ALTHALMASAT was the hidden word.
Only the H was absent.
The 3 A's, 2 C's, 2 T's, an M, and an S were all present
.
I noticed that four of the digits (1, 2:, 3, 4) were on one side of the ribbon.
This digits comprise the Pythagorean tetraktys,
which sums to 10, suggesting a "10 letter word".

On the other side of the ribbon, the digits (1, 4:) , sum to 5,
sugesting a "5 letter word."

In addition, two of the numbers, (2: and 4:)
were followed by colons.
Also, two of the letters in the ribbon were followed by colons,
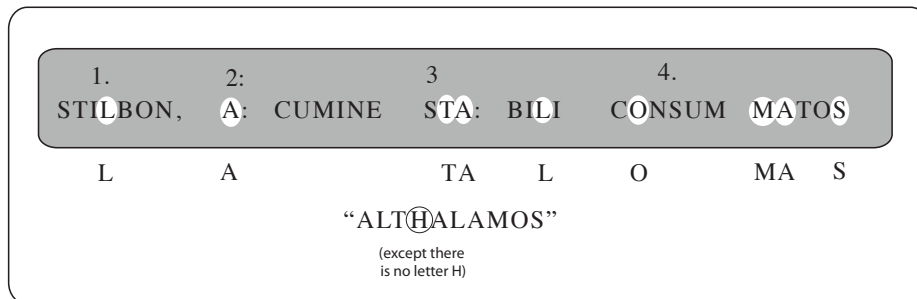and curiously they were both A's (A: and A:)

Could there be two hidden words?
One with 10 letters and another with 5 letters?

Knowing the *Monas* was published in 1564,
six years before the *Preface to Euclid*, it occurred to me
that Dee might have used a different form of the word ALTHALMASAT
(plus, I was also aware that Dee didn't give away clues that easily).

Without the fancy suffix "SAT" (similar to the "zat" which Dee used in Sabbatizat),
the word becomes ALTHALMOS, with only 9 letters.

But, Dee had discarded the "A" (from between the L and the M) in "THAL**A**MOS"
when he created ALTHALMASAT,
probably because it sounds better than the choppy "ALTHAL**A**MASAT"
(which seems "over-burdened" with A's if it includes all 5 of them).
So, Dee seemed to be hiding the 10 letter word **ALTHALAMOS.**

(the 1, 2:, 3, 4., written along one side of the "flowing ribbon")

1.	2:		3		4.	
STILBON,	A:	CUMINE	STA:	BILI	CONSUM	MATOS
L	A		TA	L	O	MA S

"ALT(H)ALAMOS"
(except there
is no letter H)

Can you figure out the 5 letter word Dee is hiding?

(the 1, 4:, written along the other side of the "flowing ribbon")

| STILBON, | A: | CUMINE | STA: | BILI | CONSUM | MATOS |
| | | | 1. | | 4: | |
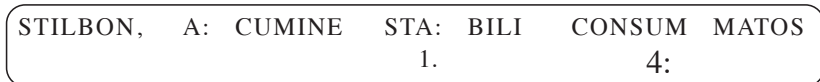
It contains "one letter" from "STABILI" and 4 letters from "CONSUMMATUS."
I'll give you some hints:
They are at the heart of Dee's philosophy about "Space and Void,"
the closest packing of spheres
and the early Greek's philosophy of what the world was made from,
and they're very small.

The anwer: **atoms**
Dee used a Greek root word for "althalamos,"
so it makes sense that he would be hiding the Greek word for "atoms,"
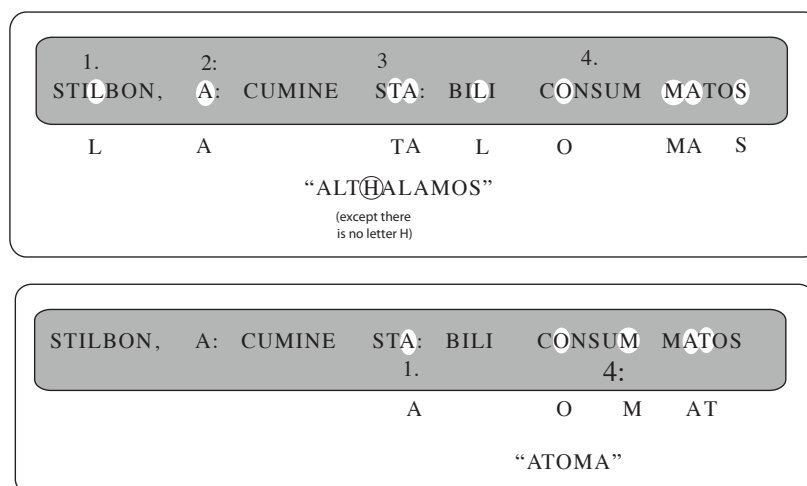**ATOMA.**

The first "**a**" in "**a**toma" means "**not**" and "toma" means "**cut**."
The Greeks felt the atom was the
teensy-weensy, indivisible, "**uncuttable**" particle
out of which everything was made.

These two words, "**althalamos**" and "**atmos**" speak loudly of Dee's philosophy,
but could also get Dee in trouble with those who didn't find
"moving images in dark rooms"
and "atomism" theologically acceptable ideas.
They summarize main themes of the Monas and feel right – except for one problem.

There's still no H.
And those curiously placed colons (:) are as yet unaccounted for.
Here's a full inventory of where the letters are found:

1.	2:		3		4.	
STILBON,	A: CUMINE		STA:	BILI	CONSUM	MATOS
L	A		TA	L	O	MA S

"ALT(H)ALAMOS"

(except there
is no letter H)

STILBON,	A: CUMINE	STA:	BILI	CONSUM	MATOS
	1.			4:	
	A			O M	AT

"ATOMA"

Curiously, both of the "A's with colons" are used in the making of "althalamos."
That darned H seems as though it should be in the word
"A: CUMINE" (sharp point), but it's nowhere to be found.
However, this **is** one of the words that contains an "A with a colon."

I didn't have to turn many pages of the Liddell-Scott Greek dictionary
to find out the definition of "A."

Strangely enough, as a prefix the letter "A" can mean **2 completely opposite things**,
which Greek grammarians refer to as "**alpha privatum**" and "**alpha copulatum**."
(Dee would obviously have been excited about this linguistic example of oppositeness.)

The "**a**" prefix in **alpha privatum** is like our prefix "**un**-" or "**not**."
It denotes negation, in the sense of not having a certain property.

The Latin word *privare* means "deprive," and *privatus* means
"withdrawn from public life," like our word "private."

It can express "want," "absence," or "without,"
as in our words atypical (not typical),
anonymous (not named), atheist (non religious),
and, of course, atom (not cuttable).

But if the prefix "**a**" is **alpha copulativum,** it has the exact opposite effect,
This type of "**a**" expresses a "**union**,
a joining together, being connected, or likeness."

Here are several examples:
Loxos means "to lie." *Aloxa* means "to lie together, bed-fellow, spouse."
Delphos means "womb." *Adelphos* means children from the same mother (brothers and sisters).
Talanton means "a pair of balance scales." *Atalantos* means "equivalent" or "equal in weight."

I wondered,
"If an atom is a "non-cuttable" point,
what might be "union" or 'a joining together" be (with with regards to a point)?"
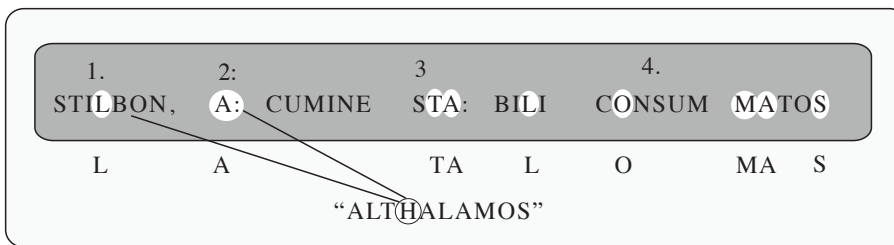Maybe 2 points?

There certainly was a very graphic depiction of two points nearby on the Title page.
The two tips of the Mercuries' spears meet at the "camera obscura hole."

This brought to mind that the word Mercurius, "that changeable thing,"
found in this "round" sentence on the flowing ribbons,
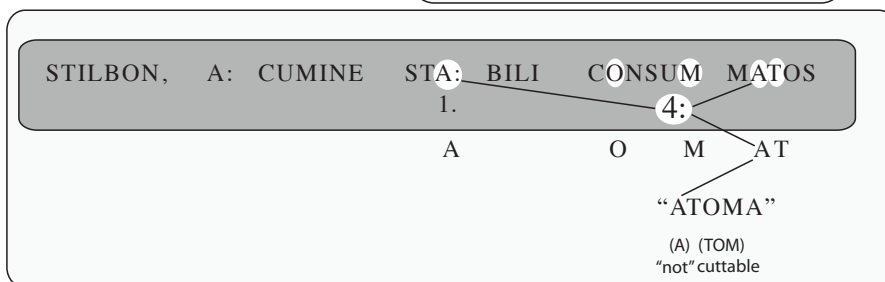which expresses Mercury as 8, or 9, or 10.

Dee had even "changed" the word from the
Latin *Mercurius* to the early Greek word *STILBON*,
(making it the only Greek word in the otherwise Latin sentence).

What would another likeness of Mercurius or STILBON be?
The answer is **Hermes**, the Greek god synonymous with STILBON
and also synonymous with the Roman god Mercurius!

Suddenly, that missing **H** has appeared!

Here, the colon signifies "alpha copulativum" in which the prefix "A" means "union", or "likeness."

A "likeness" of Stilbon (or Mercury) is Hermes, which starts with that "missing" letter, H.

1.	2:		3		4.	
STILBON,	A:	CUMINE	STA:	BILI	CONSUM	MATOS
L	A		TA	L	O	MA	S

"ALTHALAMOS"

To summarize,
Dee added colons
to the two A's
to suggest that one was
"alpha privitum"
and the other was
"alpha copulativum."

Here, the colon signifies "alpha privatum," in which the prefix "A" means "un-" or "not."

STILBON, A: CUMINE STA: BILI CONSUM MATOS

1. 4:

A   O   M   AT

"ATOMA"

(A) (TOM)
"not" cuttable

The overarching clue here is that Dee saw the behavior of light
in an althalamos (a camera obscura, a dark room)
as expressing "**oppositeness**" in Nature.
He also saw "atoma" as spheres close-packing
in the cuboctahedron as expressing "**oppositeness**" in Nature.

But, there's another subtle clue going on here.
Dee saw the letter "A" as something very special.
In the "36 Boxes chart," he portrays "alpha" (the Greek version of A)
as "principium," the "first."

Here he has "highlighted" the "letter A" twice in the words
A: CUMINE   STA: BILI, meaning a "sharp point" that is "stable."

In Theorem 10, he calls the central point of the Aries symbol
"**Acioaedes, Acuminataque**" (sharp, pointed).

In the letter to Maximillian, he says the Monas symbol
is made from the astrological symbol of mercury,
"**Acumine quodam praemunito**" or "Fortified by a Sharp Point."
(Dee, *Monas*, p. 3 verso)

The astrological
sign of Mercury...

...fortified by
a sharp point

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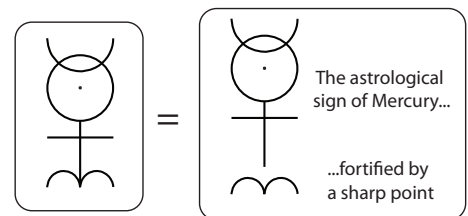
As he highlighted the two occurrences
of the letter A in "A: CUMINE STA: BILI,"
he seems to be suggesting letter A
is synonymous with a "geometrical point"
or sharp tip, a thorn tip or the tip of a bee's stinger.

$$A = \cdot$$
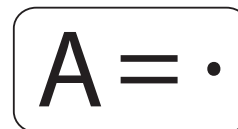
All these words Acumine, Acioaedes, Athalamos, and Atoma begin with the letter A.
In Dee's mind, they were self-referential words.
They have "pointy" first letters and their meaning involves a "point."

We can start to get an idea of what Dee is referring to
in his Letter to Maximillian about the "Art of Grammar,"
and why he declares that "alphabetic lettering contains such great Mysteries,"
and why his **A**deptius in the Tree of Rarity diagram is a **one**-in-a-million kind of guy.
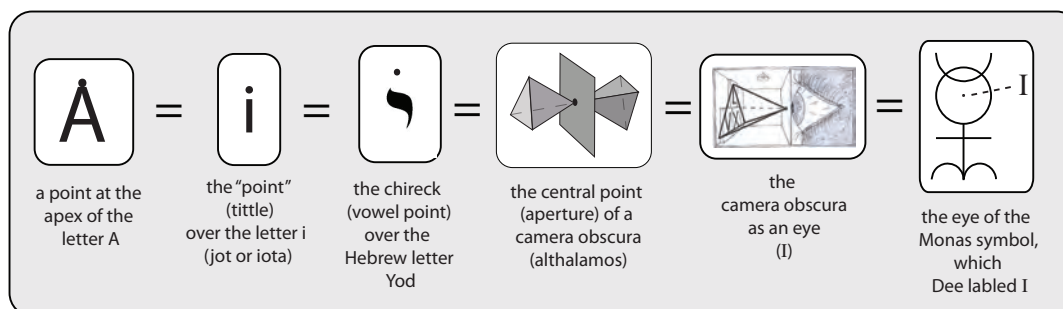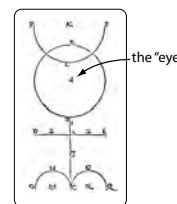(Dee, *Monas*, p. 4 verso)

We've seen his fascination with the letter X and how,
with its friends L and V, it makes LUX, or light.

We've seen in the "36 Boxes chart" that T and M representhe process of separatio and conjunctio.
It should be noted that the words AL**T**HAL**M**OS and A**TOM** each contain a **T** and an **M**.

These two power-packed words express the concept of "point"
they also each express **separatio** (sort of like alpha privatum, "un- or not")
and **conjunctio** (sort of like alpha copulativum, "a joining.")

This "point" is the same point that Dee sees
on the top of the small Greek letter *iota*
or the small Latin letter "i"
or the Hebrew *iod* with the Chireck (vowel-accent point) on top.
(in the *Letter to Maximillian*, p.5)

A homonym of this "i" is "eye,"
An eye acts like the small hole in the camera obscura.
This is the "I" marking the center of the Sun circle of the Monas symbol,
the eye of this cleverly designed cyclops-like homunculus.

the "eye"

Å	=	i	=	৲	=	◄►	=	◁	=	☿ I
a point at the apex of the letter A		the "point" (tittle) over the letter i (jot or iota)		the chireck (vowel point) over the Hebrew letter Yod		the central point (aperture) of a camera obscura (althalamos)		the camera obscura as an eye (I)		the eye of the Monas symbol, which Dee labled I

Most people don't give it much thought, but the great geometer Dee
loved philosophizing about the concept of the "point."

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